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VOL. 2

SIGHT REDUCTION TABLES

FOR

MARINE NAVIGATION

LATITUDES 15°–30°, Inclusive

PREFACE

This six-volume series of *Sight Reduction Tables for Marine Navigation* is designed to facilitate the practice of celestial navigation at sea by the Marcq Saint Hilaire or intercept method.

The tabular data are the solutions of the navigational triangle of which two sides and the included angle are known and it is necessary to find the values of the third side and adjacent angle.

The tables, intended for use with *The Nautical Almanac*, are designed for precise interpolation of altitude for declination by means of interpolation tables which facilitate linear interpolation and provide additionally for the effect of second differences when required.

The concept, design, development, and preparation of these tables are the results of the collaborative efforts and joint accomplishments of the National Imagery and Mapping Agency, the U.S. Naval Observatory, and Her Majesty's Nautical Almanac Office, Royal Greenwich Observatory. The tabular material in identical format has been published in the United Kingdom by the Hydrographic Department, Ministry of Defence (Navy), as N.P. 401.

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Users should refer corrections, additions, and comments for improving this product to:

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INTRODUCTION

A. DESCRIPTION OF TABLES

1. Purpose and Scope. The main purpose of these tables is to facilitate the practice of celestial navigation at sea. A secondary purpose is to provide, within the limitations of the tabular precision and interval, a table of the solutions of a spherical triangle of which two sides and the included angle are known and it is necessary to find the values of the third side and adjacent angle.

The tables have been designed primarily for use with the Marcq Saint Hilaire or intercept method of sight reduction, utilizing a position assumed or chosen so that interpolation for latitude and local hour angle is not required.

For entering arguments of integral degrees of latitude, declination, and local hour angle, altitudes and their differences are tabulated to the nearest tenth of a minute, azimuth angles to the nearest tenth of a degree. But the tables are designed for precise interpolation of altitude for declination only by means of interpolation tables which facilitate linear interpolation and provide additionally for the effect of second differences.

The data are applicable to the solutions of sights of all celestial bodies; there are no limiting values of altitude, latitude, hour angle, or declination.

2. Arrangement. The tables are divided into six volumes, each of which includes two eight-degree zones of latitude. An overlap of 1° occurs between volumes. The six volumes cover latitude bands 0° to 15°, 15° to 30°, 30° to 45°, 45° to 60°, 60° to 75°, and 75° to 90°.

Each consecutive opening of the pages of a latitude zone differs from the preceding one by 1° of local hour angle (LHA). As shown in figures 1 and 2, the values of LHA are prominently displayed at the top and bottom of each page; the horizontal argument heading each column is latitude, and the vertical argument is declination.

For each combination of arguments, the tabulations are: the tabular altitude (ht or Tab. Hc), the altitude difference (d) with its sign, and the azimuth angle (Z).

Within each opening, the data on the left-hand page are the altitudes, altitude differences, and azimuth angles of celestial bodies when the latitude of the observer has the same name as the declinations of the bodies. For any LHA tabulated on a left-hand page and any combination of the tabular latitude and declination arguments, the tabular altitude and associated azimuth angle respondents on the left-hand page are those of a body above the celestial horizon of the observer.

The LHA's tabulated on the left-hand pages are limited to the following ranges: 0° increasing to 90° and 360° decreasing to 270°. On any left-hand page there are two tabulated LHA's, one LHA in the range 0° increasing to 90° and the second in the range 360° decreasing to 270°.

On the right-hand page of each opening, the data above the horizontal rules are the tabular altitudes, altitude differences, and azimuth angles of celestial bodies above the celestial horizon when the latitude of the observer has a name contrary to the name of the declinations of the bodies and the LHA's of the bodies are those tabulated at the top of the page. The data below the horizontal rules are the tabular altitudes, altitude differences, and azimuth angles of celestial bodies above the celestial horizon when the latitude of the observer has the same name as the declinations of the bodies and the LHA's of the bodies are those tabulated at the bottom of the page.

The LHA's tabulated at the top of a right-hand page are the same as those tabulated on the left-hand page of the opening. The LHA's tabulated at the bottom of the right-hand page are limited to the range 90° increasing to 270°; one of the two LHA's at the bottom of the page is in the range 90° increasing to 180°; the other LHA is in the range 180° increasing to 270°; the LHA in the range 90° increasing to 180° is the supplement of the LHA at the top of the page in the range 0° increasing to 90°. When the LHA is 90°, the left and right-hand pages are identical.

The horizontal rules, known as the Contrary-Same Line or C-S Line, indicate the degree of declination in which the celestial horizon occurs.

60°, 300° L.H.A. LATITUDE SAME NAME AS DECLINATION

N. Lat. { L.H.A. greater than 180° .....Zn=Z
{ L.H.A. less than 180°............Zn=360°–Z

Dec.	15°	16°	17°	18°	19°	20°	21°	22°	Dec.
°	Hc d Z	Hc d Z	Hc d Z	Hc d Z	Hc d Z	Hc d Z	Hc d Z	Hc d Z	°
0	28 52.7 +17.5 98.5	28 43.6 +18.6 99.0	28 33.9 +19.7 99.6	28 23.6 +20.9 100.1	28 12.8 +21.9 100.6	28 01.5 +23.0 101.2	27 49.6 +24.0 101.7	27 37.1 +25.2 102.2	0
1	29 10.2 +16.9 97.4	29 02.2 +18.1 98.0	28 53.6 +19.2 98.5	28 44.5 +20.3 99.0	28 34.7 +21.5 99.6	28 24.5 +22.5 100.1	28 13.6 +23.7 100.7	28 02.3 +24.7 101.2	1
2	29 27.1 +16.4 96.3	29 20.3 +17.5 96.9	29 12.8 +18.7 97.4	29 04.8 +19.8 98.0	28 56.2 +20.9 98.5	28 47.0 +22.0 99.1	28 37.3 +23.1 99.6	28 27.0 +24.2 100.1	2
3	29 43.5 +15.9 95.2	29 37.8 +17.0 95.8	29 31.5 +18.1 96.3	29 24.6 +19.2 96.9	29 17.1 +20.4 97.4	29 09.0 +21.5 98.0	29 00.4 +22.6 98.6	28 51.2 +23.7 99.1	3
4	29 59.4 +15.2 94.1	29 54.8 +16.4 94.7	29 49.6 +17.6 95.2	29 43.8 +18.8 95.8	29 37.5 +19.9 96.4	29 30.5 +21.0 96.9	29 23.0 +22.1 97.5	29 14.9 +23.2 98.0	4
5	30 14.6 +14.7 93.0	30 11.2 +15.9 93.5	30 07.2 +17.0 94.1	30 02.6 +18.2 94.7	29 57.4 +19.3 95.3	29 51.5 +20.5 95.9	29 45.1 +21.6 96.4	29 38.1 +22.7 97.0	5
6	30 29.3 +14.1 91.8	30 27.1 +15.3 92.4	30 24.2 +16.5 93.0	30 20.8 +17.6 93.6	30 16.7 +18.8 94.2	30 12.0 +19.9 94.8	30 06.7 +21.1 95.3	30 00.8 +22.2 95.9	6
7	30 43.4 +13.6 90.7	30 42.4 +14					+20.5 94.3	30 23.0 +21.7 94.8	7
8	30 57.0 +12.9 89.6	30 57.1 +14					+20.0 93.2	30 44.7 +21.1 93.8	8
9	31 09.9 +12.3 88.4	31 11.2 +13					+19.4 92.1	31 05.8 +20.6 92.7	9
10	31 22.2 +11.7 87.3	31 24.7 +12					+18.8 91.0	31 26.4 +20.0 91.6	10
11	31 33.9 +11.1 86.1	31 37.6 +12					+18.3 89.8	31 46.4 +19.4 90.5	11
12	31 45.0 +10.5 85.0	31 49.9 +11					+17.6 88.7	32 05.8 +18.8 89.4	12
13	31 55.5 +9.8 83.8	32 01.6 +11					+17.1 87.6	32 24.6 +18.2 88.2	13
14	32 05.3 +9.2 82.7	32 12.7 +10					+16.4 86.5	32 42.8 +17.7 87.1	14
15	32 14.5 +8.6 81.5	32 23.1 +9					+15.8 85.3	33 00.5 +16.9 86.0	15
16	32 23.1 +7.8 80.3	32 32.8 +9					+15.1 84.2	33 17.4 +16.4 84.8	16
17	32 30.9 +7.3 79.2	32 41.9 +8					+14.5 83.0	33 33.8 +15.7 83.7	17
18	32 38.2 +6.6 78.0	32 50.4 +7					+13.9 81.8	33 49.5 +15.1 82.5	18
19	32 44.8 +5.9 76.8	32 58.1 +7					+13.2 80.7	34 04.6 +14.4 81.3	19
20	32 50.7 +5.2 75.6	33 05.2 +6					+12.5 79.5	34 19.0 +13.7 80.2	20
21	32 55.9 +4.6 74.4	33 11.7 +5					+11.8 78.3	34 32.7 +13.0 79.0	21
22	33 00.5 +3.8 73.2	33 17.4 +5					+11.1 77.1	34 45.7 +12.3 77.8	22
23	33 04.3 +3.3 72.0	33 22.5 +4					+10.4 75.9	34 58.0 +11.7 76.6	23
24	33 07.6 +2.5 70.9	33 26.9 +3					+9.7 74.7	35 09.7 +10.9 75.4	24
25	33 10.1 +1.8 69.7	33 30.6 +3					+9.0 73.5	35 20.6 +10.2 74.2	25
26	33 11.9 +1.2 68.5	33 33.6 +2					+8.3 72.3	35 30.8 +9.5 73.0	26
27	33 13.1 +0.4 67.3	33 36.0 +1					+7.5 71.1	35 40.3 +8.7 71.8	27
28	33 13.5 −0.2 66.1	33 37.6 +0					+6.8 69.9	35 49.0 +8.0 70.6	28
29	33 13.3 −0.9 64.9	33 38.5 +0					+6.1 68.7	35 57.0 +7.3 69.3	29
30	33 12.4 −1.6 63.7	33 38.7 −0					+5.3 67.4	36 04.3 +6.5 68.1	30
31	33 10.8 −2.2 62.5	33 38.3 −1					+4.5 66.2	36 10.8 +5.7 66.9	31
32	33 08.6 −3.0 61.3	33 37.1 −1					+3.8 65.0	36 16.5 +5.0 65.6	32
33	33 05.6 −3.6 60.1	33 35.3 −2					+3.1 63.7	36 21.5 +4.2 64.4	33
34	33 02.0 −4.3 58.9	33 32.7 −3					+2.3 62.5	36 25.7 +3.4 63.2	34
35	32 57.7 −5.0 57.7	33 29.5 −4					+1.5 61.3	36 29.1 +2.7 61.9	35
36	32 52.7 −5.7 56.5	33 25.5 −4					+0.8 60.0	36 31.8 +1.9 60.7	36
37	32 47.0 −6.3 55.4	33 20.9 −5					0.0 58.8	36 33.7 +1.1 59.4	37
38	32 40.7 −7.0 54.2	33 15.6 −6					−0.7 57.6	36 34.8 +0.3 58.2	38
39	32 33.7 −7.6 53.0	33 09.6 −6					−1.6 56.3	36 35.1 −0.4 56.9	39
40	32 26.1 −8.3 51.8	33 03.0 −7					−2.2 55.1	36 34.7 −1.2 55.7	40
41	32 17.8 −9.0 50.6	32 55.6 −8					−3.1 53.9	36 33.5 −2.0 54.5	41
42	32 08.8 −9.6 49.5	32 47.6 −8					−3.7 52.6	36 31.5 −2.7 53.2	42
43	31 59.2 −10.2 48.3	32 39.0 −9					−4.6 51.4	36 28.8 −3.6 52.0	43
44	31 49.0 −10.8 47.1	32 29.6 −9					−5.3 50.2	36 25.2 −4.3 50.7	44
45	31 38.2 −11.5 46.0	32 19.7 −10					−6.0 48.9	36 20.9 −5.1 49.5	45
46	31 26.7 −12.1 44.8	32 09.1 −11					−6.8 47.7	36 15.8 −5.8 48.3	46
47	31 14.6 −12.7 43.7	31 57.8 −11					−7.5 46.5	36 10.0 −6.6 47.0	47
48	31 01.9 −13.3 42.6	31 45.9 −12					−8.2 45.3	36 03.4 −7.3 45.8	48
49	30 48.6 −13.9 41.4	31 33.4 −13					−9.0 44.1	35 56.1 −8.1 44.6	49
50	30 34.7 −14.5 40.3	31 20.3 −13.7 40.7	32 05.7 −13.0 41.1	32 50.8 −12.2 41.5	33 35.6 −11.4 41.9	34 20.0 −10.5 42.4	35 04.2 −9.7 42.9	35 48.0 −8.8 43.3	50
51	30 20.2 −15.1 39.2	31 06.6 −14.3 39.5	31 52.7 −13.5 39.9	32 38.6 −12.8 40.3	33 24.2 −12.0 40.8	34 09.5 −11.2 41.2	34 54.5 −10.4 41.7	35 39.2 −9.6 42.1	51
52	30 05.1 −15.6 38.0	30 52.3 −14.9 38.4	31 39.2 −14.2 38.8	32 25.8 −13.4 39.2	33 12.2 −12.7 39.6	33 58.3 −11.9 40.0	34 44.1 −11.1 40.5	35 29.6 −10.3 40.9	52
53	29 49.5 −16.2 36.9	30 37.4 −15.5 37.3	31 25.0 −14.8 37.6	32 12.4 −14.1 38.0	32 59.5 −13.3 38.4	33 46.4 −12.6 38.8	34 33.0 −11.8 39.3	35 19.3 −11.0 39.7	53
54	29 33.3 −16.7 35.8	30 21.9 −16.1 36.2	31 10.2 −15.3 36.5	31 58.3 −14.6 36.9	32 46.2 −13.9 37.3	33 33.8 −13.2 37.7	34 21.2 −12.5 38.1	35 08.3 −11.7 38.5	54
55	29 16.6 −17.3 34.7	30 05.8 −16.6 35.0	30 54.9 −16.0 35.4	31 43.7 −15.3 35.7	32 32.3 −14.6 36.1	33 20.6 −13.9 36.5	34 08.7 −13.1 36.9	34 56.6 −12.4 37.3	55
56	28 59.3 −17.8 33.6	29 49.2 −17.2 33.9	30 38.9 −16.5 34.3	31 28.4 −15.9 34.6	32 17.7 −15.2 35.0	33 06.7 −14.5 35.3	33 55.6 −13.8 35.7	34 44.2 −13.1 36.1	56
57	28 41.5 −18.3 32.5	29 32.0 −17.7 32.8	30 22.4 −17.1 33.1	31 12.5 −16.5 33.5	32 02.5 −15.9 33.8	32 52.2 −15.1 34.2	33 41.8 −14.5 34.5	34 31.1 −13.8 34.9	57
58	28 23.2 −18.8 31.4	29 14.3 −18.2 31.7	30 05.3 −17.7 32.0	30 56.0 −17.0 32.3	31 46.6 −16.4 32.7	32 37.1 −15.8 33.0	33 27.3 −15.2 33.4	34 17.3 −14.5 33.7	58
59	28 04.4 −19.4 30.4	28 56.1 −18.8 30.6	29 47.6 −18.2 30.9	30 39.0 −17.6 31.2	31 30.2 −17.0 31.5	32 21.3 −16.4 31.9	33 12.1 −15.8 32.2	34 02.8 −15.1 32.6	59
60	27 45.0 −19.8 29.3	28 37.3 −19.3 29.6	29 29.4 −18.7 29.8	30 21.4 −18.2 30.1	31 13.2 −17.6 30.4	32 04.9 −17.0 30.7	32 56.4 −16.4 31.1	33 47.7 −15.8 31.4	60
61	27 25.2 −20.3 28.2	28 18.0 −19.8 28.5	29 10.7 −19.3 28.7	30 03.2 −18.7 29.0	30 55.6 −18.1 29.3	31 47.9 −17.6 29.6	32 40.0 −17.1 29.9	33 31.9 −16.5 30.2	61
62	27 04.9 −20.8 27.2	27 58.2 −20.3 27.4	28 51.4 −19.7 27.7	29 44.5 −19.2 27.9	30 37.5 −18.8 28.2	31 30.3 −18.2 28.5	32 22.9 −17.6 28.8	33 15.4 −17.0 29.1	62
63	26 44.1 −21.2 26.1	27 37.9 −20.7 26.3	28 31.7 −20.3 26.6	29 25.3 −19.8 26.8	30 18.7 −19.2 27.1	31 12.1 −18.8 27.4	32 05.3 −18.2 27.6	32 58.4 −17.7 27.9	63
64	26 22.9 −21.7 25.1	27 17.2 −21.3 25.3	28 11.4 −20.8 25.5	29 05.5 −20.3 25.8	29 59.5 −19.9 26.0	30 53.3 −19.3 26.3	31 47.1 −18.8 26.5	32 40.7 −18.3 26.8	64
65	26 01.2 −22.2 24.0	26 55.9 −21.7 24.2	27 50.6 −21.3 24.5	28 45.2 −20.8 24.7	29 39.6 −20.3 24.9	30 34.0 −19.9 25.2	31 28.3 −19.4 25.4	32 22.4 −18.9 25.7	65
66	25 39.0 −22.5 23.0	26 34.2 −22.1 23.2	27 29.3 −21.7 23.4	28 24.4 −21.3 23.6	29 19.3 −20.9 23.8	30 14.1 −20.4 24.1	31 08.9 −20.0 24.3	32 03.5 −19.5 24.6	66
67	25 16.5 −23.0 22.0	26 12.1 −22.6 22.2	27 07.6 −22.2 22.3	28 03.1 −21.8 22.5	28 58.4 −21.3 22.8	29 53.7 −20.9 23.0	30 48.9 −20.5 23.2	31 44.0 −20.1 23.4	67
68	24 53.5 −23.4 21.0	25 49.5 −23.1 21.1	26 45.4 −22.7 21.3	27 41.3 −22.3 21.5	28 37.1 −21.9 21.7	29 32.8 −21.5 21.9	30 28.4 −21.1 22.1	31 23.9 −20.6 22.3	68
69	24 30.1 −23.9 19.9	25 26.4 −23.4 20.1	26 22.7 −23.1 20.3	27 19.0 −22.7 20.4	28 15.2 −22.4 20.6	29 11.3 −22.0 20.8	30 07.3 −21.5 21.0	31 03.3 −21.2 21.2	69
70	24 06.2 −24.2 18.9	25 03.0 −23.9 19.1	25 59.6 −23.5 19.2	26 56.3 −23.2 19.4	27 52.8 −22.8 19.6	28 49.3 −22.4 19.8	29 45.8 −22.1 20.0	30 42.1 −21.7 20.2	70
71	23 42.0 −24.6 17.9	24 39.1 −24.3 18.1	25 36.1 −23.9 18.2	26 33.1 −23.7 18.4	27 30.0 −23.3 18.5	28 26.9 −23.0 18.7	29 23.7 −22.7 18.9	30 20.4 −22.3 19.1	71
72	23 17.4 −24.9 16.9	24 14.8 −24.7 17.2	25 12.2 −24.4 17.2	26 09.4 −24.0 17.3	27 06.7 −23.8 17.5	28 03.9 −23.4 17.7	29 01.0 −23.1 17.8	29 58.1 −22.7 18.0	72
73	22 52.5 −25.4 16.0	23 50.1 −25.0 16.1	24 47.8 −24.8 16.2	25 45.4 −24.5 16.3	26 42.9 −24.2 16.5	27 40.5 −23.9 16.6	28 37.9 −23.6 16.8	29 35.4 −23.3 16.9	73
74	22 27.1 −25.6 15.0	23 25.1 −25.4 15.1	24 23.0 −25.2 15.2	25 20.9 −24.9 15.3	26 18.7 −24.6 15.6	27 16.6 −24.4 15.6	28 14.3 −24.0 15.7	29 12.1 −23.8 15.9	74
75	22 01.5 −26.1 14.0	22 59.7 −25.8 14.1	23 57.8 −25.5 14.2	24 56.0 −25.3 14.3	25 54.1 −25.0 14.4	26 52.2 −24.8 14.6	27 50.3 −24.6 14.7	28 48.3 −24.3 14.8	75
76	21 35.4 −26.3 13.0	22 33.9 −26.2 13.1	23 32.3 −25.9 13.2	24 30.7 −25.7 13.3	25 29.1 −25.5 13.4	26 27.4 −25.2 13.5	27 25.7 −24.9 13.7	28 24.0 −24.7 13.8	76
77	21 09.1 −26.7 12.1	22 07.7 −26.5 12.1	23 06.4 −26.3 12.2	24 05.0 −26.1 12.3	25 03.6 −25.8 12.4	26 02.2 −25.6 12.5	27 00.8 −25.5 12.6	27 59.3 −25.2 12.7	77
78	20 42.4 −27.0 11.1	21 41.2 −26.8 11.2	22 40.1 −26.6 11.3	23 38.9 −26.4 11.3	24 37.8 −26.3 11.4	25 36.6 −26.1 11.5	26 35.3 −25.8 11.6	27 34.1 −25.6 11.7	78
79	20 15.4 −27.3 10.1	21 14.4 −27.1 10.2	22 13.5 −27.0 10.3	23 12.5 −26.8 10.4	24 11.5 −26.6 10.4	25 10.5 −26.4 10.5	26 09.5 −26.3 10.6	27 08.5 −26.1 10.7	79
80	19 48.1 −27.7 9.2	20 47.3 −27.5 9.3	21 46.5 −27.3 9.3	22 45.7 −27.2 9.4	23 44.9 −27.0 9.5	24 44.1 −26.9 9.5	25 43.2 −26.6 9.6	26 42.4 −26.5 9.7	80
81	19 20.4 −27.9 8.3	20 19.8 −27.8 8.3	21 19.2 −27.6 8.4	22 18.5 −27.5 8.4	23 17.9 −27.4 8.5	24 17.2 −27.2 8.5	25 16.6 −27.1 8.6	26 15.9 −26.9 8.7	81
82	18 52.5 −28.1 7.3	19 52.0 −28.0 7.4	20 51.6 −28.0 7.4	21 51.0 −27.8 7.5	22 50.5 −27.7 7.5	23 50.0 −27.6 7.6	24 49.5 −27.5 7.6	25 49.0 −27.4 7.7	82
83	18 24.4 −28.5 6.4	19 24.0 −28.4 6.4	20 23.6 −28.2 6.5	21 23.2 −28.1 6.5	22 22.8 −28.0 6.6	23 22.4 −27.9 6.6	24 22.0 −27.8 6.7	25 21.6 −27.7 6.7	83
84	17 55.9 −28.7 5.5	18 55.6 −28.6 5.5	19 55.3 −28.6 5.5	20 55.1 −28.5 5.6	21 54.8 −28.4 5.6	22 54.5 −28.3 5.6	23 54.2 −28.2 5.7	24 53.9 −28.1 5.7	84
85	17 27.2 −29.0 4.5	18 27.0 −28.9 4.6	19 26.8 −28.8 4.6	20 26.6 −28.7 4.6	21 26.4 −28.7 4.7	22 26.2 −28.6 4.7	23 26.0 −28.5 4.7	24 25.8 −28.4 4.8	85
86	16 58.2 −29.2 3.6	17 58.1 −29.2 3.6	18 58.0 −29.1 3.7	19 57.9 −29.1 3.7	20 57.7 −29.0 3.7	21 57.6 −28.9 3.7	22 57.5 −28.9 3.8	23 57.4 −28.9 3.8	86
87	16 29.0 −29.4 2.7	17 28.9 −29.4 2.7	18 28.9 −29.3 2.7	19 28.8 −29.3 2.8	20 28.7 −29.3 2.8	21 28.7 −29.2 2.8	22 28.6 −29.2 2.8	23 28.5 −29.1 2.8	87
88	15 59.6 −29.7 1.8	16 59.5 −29.6 1.8	17 59.5 −29.6 1.8	18 59.5 −29.6 1.8	19 59.4 −29.5 1.8	20 59.4 −29.5 1.9	21 59.4 −29.6 1.9	22 59.4 −29.6 1.9	88
89	15 29.9 −29.9 0.9	16 29.9 −29.9 0.9	17 29.9 −29.9 0.9	18 29.9 −29.9 0.9	19 29.9 −29.9 0.9	20 29.9 −29.9 0.9	21 29.8 −29.8 0.9	22 29.8 −29.8 0.9	89
90	15 00.0 −30.1 0.0	16 00.0 −30.1 0.0	17 00.0 −30.1 0.0	18 00.0 −30.1 0.0	19 00.0 −30.1 0.0	20 00.0 −30.1 0.0	21 00.0 −30.1 0.0	22 00.0 −30.2 0.0	90
	15°	16°	17°	18°	19°	20°	21°	22°	

60°, 300° L.H.A.   LATITUDE SAME NAME AS DECLINATION

FIGURE 1
VI

LATITUDE CONTRARY NAME TO DECLINATION L.H.A. 60°, 300°

Dec.	15° Hc	d	Z	16° Hc	d	Z	17° Hc	d	Z	18° Hc	d	Z	19° Hc	d	Z	20° Hc	d	Z	21° Hc	d	Z	22° Hc	d	Z	Dec.
0	28 52.7	−18.0	98.5	28 43.6	−19.1	99.0	28 33.9	−20.2	99.6	28 23.6	−21.3	100.1	28 12.8	−22.4	100.6	28 01.5	−23.5	101.2	27 49.6	−24.6	101.7	27 37.1	−25.5	102.2	0
1	28 34.7	−18.5	99.6	28 24.5	−19.6	100.1	28 13.7	−20.8	100.7	28 02.3	−21.8	101.2	27 50.4	−22.9	101.7	27 38.0	−24.0	102.2	27 25.0	−25.0	102.7	27 11.6	−26.1	103.2	1
2	28 16.2	−19.0	100.7	28 04.9	−20.2	101.2	27 52.9	−21.2	101.7	27 40.5	−22.3	102.2	27 27.5	−23.3	102.7	27 14.0	−24.4	103.2	27 00.0	−25.4	103.7	26 45.5	−26.4	104.2	2
3	27 57.2	−19.5	101.7	27 44.7	−20.6	102.3	27 31.7	−21.7	102.8	27 18.2	−22.7	103.3	27 04.2	−23.8	103.8	26 49.6	−24.8	104.3	26 34.6	−25.9	104.8	26 19.1	−26.9	105.2	3
4	27 37.7	−20.0	102.8	27 24.1	−21.1	103.3	27 10.0	−22.1	103.8	26 55.5	−23.2	104.3	26 40.4	−24.3	104.8	26 24.8	−25.3	105.3	26 08.7	−26.2	105.8	25 52.2	−27.3	106.2	4
5	27 17.7	−20.5	103.9	27 03.0	−21.5	104.4	26 47.9	−22.6	104.9	26 32.3	−23.7	105.3	26 16.1	−24.6	105.8	25 59.5	−25.7	106.3	25 42.5	−26.7	106.8	25 24.9	−27.6	107.2	5
6	26 57.2	−21.0	104.9	26 41.5	−22.0	105.4	26 25.3	−23.1	105.9	26 08.6	−24.1	106.4	25 51.5	−25.1	106.8	25 33.8	−26.0	107.3	25 15.8	−27.1	107.8	24 57.3	−28.1	108.2	6
7	26 36.2	−21.4	106.0	26 19.5	−22.5	106.5	26 02.2	−23.4	106.9	25 44.5	−24.5	107.4	25 26.4	−25.5	107.8	25 07.8	−26.5	108.3	24 48.7	−27.5	108.7	24 29.2	−28.4	109.2	7
8	26 14.8	−21.8	107.0	25 57.0	−22.9															−27.8	109.7	24 00.8	−28.8	110.1	8
9	25 53.0	−22.3	108.1	25 34.1	−23.3															−28.2	110.7	23 32.0	−29.1	111.1	9
10	25 30.7	−22.8	109.1	25 10.8	−23.7															−28.5	111.7	23 02.9	−29.5	112.1	10
11	25 07.9	−23.1	110.1	24 47.1	−24.2															−28.9	112.6	22 33.4	−29.8	113.0	11
12	24 44.8	−23.6	111.1	24 22.9	−24.5															−29.3	113.6	22 03.6	−30.1	113.9	12
13	24 21.2	−24.0	112.1	23 58.4	−24.9															−29.5	114.5	21 33.5	−30.5	114.9	13
14	23 57.2	−24.3	113.1	23 33.5	−25.3															−29.9	115.4	21 03.0	−30.7	115.8	14
15	23 32.9	−24.7	114.1	23 08.2	−25.7															−30.1	116.4	20 32.3	−31.0	116.7	15
16	23 08.2	−25.1	115.1	22 42.5	−26.0															−30.5	117.3	20 01.3	−31.3	117.6	16
17	22 43.1	−25.5	116.1	22 16.5	−26.4															−30.7	118.2	19 30.0	−31.5	118.5	17
18	22 17.6	−25.8	117.1	21 50.1	−26.7															−31.0	119.1	18 58.5	−31.9	119.4	18
19	21 51.8	−26.2	118.1	21 23.4	−27.1															−31.3	120.0	18 26.6	−32.0	120.3	19
20	21 25.6	−26.5	119.0	20 56.3	−27.3															−31.5	120.9	17 54.6	−32.4	121.2	20
21	20 59.1	−26.8	120.0	20 29.0	−27.7															−31.8	121.8	17 22.2	−32.5	122.1	21
22	20 32.3	−27.1	121.0	20 01.3	−28.0															−32.0	122.7	16 49.7	−32.8	123.0	22
23	20 05.2	−27.4	121.9	19 33.3	−28.2															−32.2	123.6	16 16.9	−33.0	123.9	23
24	19 37.8	−27.7	122.9	19 05.1	−28.6															−32.4	124.5	15 43.9	−33.1	124.7	24
25	19 10.1	−28.1	123.8	18 36.5	−28.8															−32.7	125.4	15 10.8	−33.4	125.6	25
26	18 42.0	−28.2	124.7	18 07.7	−29.0															−32.8	126.2	14 37.4	−33.6	126.4	26
27	18 13.8	−28.6	125.7	17 38.7	−29.4															−33.1	127.1	14 03.8	−33.7	127.3	27
28	17 45.2	−28.8	126.6	17 09.3	−29.5															−33.2	128.0	13 30.1	−34.0	128.2	28
29	17 16.4	−29.1	127.5	16 39.8	−29.8															−33.4	128.8	12 56.1	−34.1	129.0	29
30	16 47.3	−29.3	128.4	16 10.0	−30.1															−33.6	129.7	12 22.0	−34.2	129.8	30
31	16 18.0	−29.5	129.3	15 39.9	−30.2															−33.7	130.5	11 47.8	−34.4	130.7	31
32	15 48.5	−29.7	130.2	15 09.7	−30.5															−33.9	131.4	11 13.4	−34.5	131.5	32
33	15 18.8	−30.0	131.1	14 39.2	−30.7															−34.0	132.2	10 38.9	−34.7	132.4	33
34	14 48.8	−30.2	132.0	14 08.5	−30.8															−34.1	133.0	10 04.2	−34.8	133.2	34
35	14 18.6	−30.4	132.9	13 37.7	−31.1															−34.3	133.9	9 29.4	−34.9	134.0	35
36	13 48.2	−30.6	133.8	13 06.6	−31.2															−34.4	134.7	8 54.5	−35.0	134.8	36
37	13 17.6	−30.7	134.7	12 35.4	−31.4															−34.5	135.5	8 19.5	−35.1	135.7	37
38	12 46.9	−30.9	135.6	12 04.0	−31.6															−34.6	136.4	7 44.4	−35.2	136.5	38
39	12 16.0	−31.1	136.5	11 32.4	−31.7															−34.7	137.2	7 09.2	−35.3	137.3	39
40	11 44.9	−31.3	137.3	11 00.7	−31.9															−34.8	138.0	6 33.9	−35.3	138.1	40
41	11 13.6	−31.4	138.2	10 28.8	−32.0															−34.9	138.8	5 58.6	−35.3	138.9	41
42	10 42.2	−31.5	139.1	9 56.8	−32.1															−35.0	139.7	5 23.1	−35.5	139.7	42
43	10 10.7	−31.7	139.9	9 24.7	−32.3															−35.0	140.5	4 47.6	−35.5	140.5	43
44	9 39.0	−31.8	140.8	8 52.4	−32.3															−35.1	141.3	4 12.1	−35.6	141.3	44
45	9 07.2	−32.0	141.7	8 20.1	−32.5															−35.1	142.1	3 36.5	−35.7	142.2	45
46	8 35.2	−32.0	142.5	7 47.6	−32.6															−35.2	142.9	3 00.8	−35.7	143.0	46
47	8 03.2	−32.2	143.4	7 15.0	−32.7															−35.2	143.7	2 25.1	−35.7	143.8	47
48	7 31.0	−32.2	144.2	6 42.3	−32.7															−35.3	144.5	1 49.4	−35.7	144.6	48
49	6 58.8	−32.4	145.1	6 09.6	−32.9															−35.3	145.4	1 13.7	−35.8	145.4	49
50	6 26.4	−32.4	145.9	5 36.7	−32.9															−35.3	146.2	0 37.9	−35.8	146.2	50
51	5 54.0	−32.5	146.8	5 03.8	−33.0	146.8	4 13.6	−33.5	146.9	3 23.3	−33.9	146.9	2 33.0	−34.3	146.9	1 42.7	−34.8	147.0	0 52.4	−35.3	147.0	0 02.1	−35.7	147.0	51
52	5 21.5	−32.6	147.6	4 30.8	−33.0	147.7	3 40.1	−33.5	147.7	2 49.4	−34.0	147.7	1 58.7	−34.5	147.8	1 07.9	−34.9	147.8	0 17.1	−35.3	147.8	0 33.6	+35.8	32.2	52
53	4 48.9	−32.6	148.5	3 57.8	−33.1	148.5	3 06.6	−33.5	148.5	2 15.4	−33.9	148.6	1 24.2	−34.4	148.6	0 33.0	−34.8	148.6				1 09.4	+35.7	31.4	53
54	4 16.3	−32.7	149.3	3 24.7	−33.1	149.3	2 33.1	−33.6	149.4	1 41.5	−34.0	149.4	0 49.8	−34.4	149.4	0 01.8	+34.9	30.6		C-S Line (Contrary-Same Line)		1 45.1	+35.7	30.6	54
55	3 43.6	−32.7	150.1	2 51.6	−33.2	150.2	1 59.5	−33.6	150.2	1 07.5	−34.1	150.2	0 15.4	−34.5	150.2	0 36.7	+34.9	29.8				2 20.8	+35.7	29.8	55
56	3 10.9	−32.8	151.0	2 18.4	−33.2	151.0	1 25.9	−33.6	151.0	0 33.4	−34.0	151.0	0 19.1	+34.4	29.0	1 11.6	+34.8	29.0				2 56.5	+35.7	29.0	56
57	2 38.1	−32.8	151.8	1 45.2	−33.2	151.8	0 52.3	−33.6	151.9	0 00.6	+34.0	28.1	0 53.5	+34.4	28.1	1 46.4	+34.8	28.2	2 39.3	+35.2	28.2	3 32.2	+35.6	28.2	57
58	2 05.3	−32.8	152.7	1 12.0	−33.2	152.7	0 18.7	−33.6	152.7	0 34.6	+34.0	27.3	1 27.9	+34.4	27.3	2 21.2	+34.8	27.3	3 14.5	+35.2	27.4	4 07.8	+35.6	27.4	58
59	1 32.5	−32.9	153.5	0 38.8	−33.3	153.5	0 14.9	+33.7	26.5	1 08.6	+34.0	26.5	2 02.3	+34.4	26.5	2 56.0	+34.8	26.5	3 49.7	+35.1	26.6	4 43.4	+35.5	26.6	59
60	0 59.6	−32.9	154.3	0 05.5	−33.2	154.3	0 48.6	+33.6	25.7	1 42.6	+34.0	25.7	2 36.7	+34.4	25.7	3 30.8	+34.7	25.7	4 24.8	+35.1	25.7	5 18.9	+35.4	25.8	60
61	0 26.7	−32.8	155.2	0 27.7	+33.3	24.8	1 22.2	+33.6	24.8	2 16.6	+34.0	24.8	3 11.1	+34.3	24.9	4 05.5	+34.7	24.9	4 59.9	+35.0	24.9	5 54.3	+35.4	25.0	61
62	0 06.1	+32.9	24.0	1 01.0	+33.2	24.0	1 55.8	+33.5	24.0	2 50.6	+33.9	24.0	3 45.4	+34.2	24.0	4 40.2	+34.6	24.1	5 34.9	+35.0	24.1	6 29.7	+35.3	24.2	62
63	0 39.0	+32.9	23.2	1 34.2	+33.2	23.2	2 29.3	+33.6	23.2	3 24.5	+33.9	23.2	4 19.6	+34.3	23.2	5 14.8	+34.5	23.3	6 09.9	+34.9	23.3	7 05.0	+35.2	23.3	63
64	1 11.9	+32.8	22.3	2 07.4	+33.2	22.3	3 02.9	+33.5	22.3	3 58.4	+33.8	22.4	4 53.9	+34.1	22.4	5 49.3	+34.5	22.4	6 44.8	+34.8	22.5	7 40.2	+35.1	22.5	64
65	1 44.7	+32.9	21.5	2 40.6	+33.1	21.5	3 36.4	+33.4	21.5	4 32.2	+33.8	21.5	5 28.0	+34.1	21.6	6 23.8	+34.4	21.6	7 19.6	+34.7	21.7	8 15.3	+35.0	21.7	65
66	2 17.6	+32.8	20.6	3 13.7	+33.1	20.7	4 09.8	+33.4	20.7	5 06.0	+33.7	20.7	6 02.1	+34.0	20.7	6 58.2	+34.3	20.8	7 54.3	+34.6	20.9	8 50.3	+35.0	20.9	66
67	2 50.4	+32.7	19.8	3 46.8	+33.1	19.8	4 43.2	+33.4	19.8	5 39.7	+33.6	19.9	6 36.1	+33.9	19.9	7 32.5	+34.2	20.0	8 28.9	+34.5	20.0	9 25.3	+34.8	20.1	67
68	3 23.1	+32.7	19.0	4 19.9	+33.0	19.0	5 16.6	+33.3	19.0	6 13.3	+33.6	19.0	7 10.0	+33.9	19.1	8 06.7	+34.1	19.1	9 03.4	+34.4	19.2	10 00.1	+34.6	19.2	68
69	3 55.8	+32.7	18.1	4 52.9	+32.9	18.1	5 49.9	+33.2	18.2	6 46.9	+33.4	18.2	7 43.9	+33.7	18.3	8 40.8	+34.0	18.3	9 37.8	+34.3	18.3	10 34.7	+34.6	18.4	69
70	4 28.5	+32.6	17.3	5 25.8	+32.9	17.3	6 23.1	+33.1	17.3	7 20.3	+33.4	17.4	8 17.6	+33.6	17.4	9 14.8	+33.9	17.5	10 12.1	+34.1	17.5	11 09.3	+34.4	17.6	70
71	5 01.1	+32.6	16.4	5 58.7	+32.8	16.5	6 56.2	+33.0	16.5	7 53.7	+33.3	16.5	8 51.2	+33.6	16.6	9 48.7	+33.8	16.6	10 46.2	+34.0	16.7	11 43.7	+34.2	16.7	71
72	5 33.7	+32.4	15.6	6 31.5	+32.7	15.6	7 29.2	+33.0	15.7	8 27.0	+33.1	15.7	9 24.8	+33.4	15.7	10 22.5	+33.6	15.8	11 20.2	+33.9	15.8	12 17.9	+34.1	15.9	72
73	6 06.1	+32.4	14.8	7 04.2	+32.6	14.8	8 02.2	+32.8	14.8	9 00.2	+33.0	14.9	9 58.2	+33.2	14.9	10 56.1	+33.5	14.9	11 54.1	+33.7	15.0	12 52.0	+34.0	15.1	73
74	6 38.5	+32.3	13.9	7 36.8	+32.5	13.9	8 35.0	+32.7	14.0	9 33.2	+33.0	14.0	10 31.4	+33.1	14.1	11 29.6	+33.4	14.1	12 27.8	+33.6	14.2	13 26.0	+33.8	14.2	74
75	7 10.8	+32.3	13.1	8 09.3	+32.4	13.1	9 07.7	+32.6	13.1	10 06.2	+32.8	13.2	11 04.6	+33.0	13.2	12 03.0	+33.2	13.2	13 01.4	+33.4	13.3	13 59.8	+33.6	13.4	75
76	7 43.1	+32.1	12.2	8 41.7	+32.3	12.2	9 40.3	+32.5	12.3	10 39.0	+32.6	12.3	11 37.6	+32.8	12.4	12 36.2	+33.0	12.4	13 34.8	+33.2	12.4	14 33.4	+33.4	12.5	76
77	8 15.2	+32.0	11.4	9 14.0	+32.2	11.4	10 12.8	+32.4	11.4	11 11.6	+32.5	11.5	12 10.4	+32.7	11.5	13 09.2	+32.9	11.5	14 08.0	+33.0	11.6	15 06.8	+33.2	11.6	77
78	8 47.2	+31.9	10.5	9 46.2	+32.0	10.5	10 45.2	+32.2	10.6	11 44.1	+32.4	10.6	12 43.1	+32.5	10.6	13 42.1	+32.7	10.7	14 41.0	+32.9	10.7	15 40.0	+33.0	10.8	78
79	9 19.1	+31.7	9.6	10 18.2	+31.9	9.7	11 17.4	+32.0	9.7	12 16.5	+32.2	9.7	13 15.6	+32.4	9.8	14 14.8	+32.5	9.8	15 13.9	+32.6	9.9	16 13.0	+32.8	9.9	79
80	9 50.8	+31.7	8.8	10 50.1	+31.8	8.8	11 49.4	+31.9	8.8	12 48.7	+32.0	8.9	13 48.0	+32.1	8.9	14 47.3	+32.3	8.9	15 46.5	+32.5	9.0	16 45.8	+32.6	9.0	80
81	10 22.5	+31.5	7.9	11 21.9	+31.6	7.9	12 21.3	+31.7	8.0	13 20.7	+31.9	8.0	14 20.1	+32.0	8.0	15 19.6	+32.0	8.1	16 19.0	+32.2	8.1	17 18.4	+32.3	8.2	81
82	10 54.0	+31.3	7.1	11 53.5	+31.5	7.1	12 53.0	+31.6	7.1	13 52.6	+31.7	7.1	14 52.1	+31.8	7.2	15 51.6	+31.9	7.2	16 51.2	+32.0	7.2	17 50.7	+32.1	7.3	82
83	11 25.3	+31.2	6.2	12 25.0	+31.2	6.2	13 24.6	+31.4	6.2	14 24.3	+31.4	6.3	15 23.9	+31.6	6.3	16 23.5	+31.7	6.3	17 23.2	+31.7	6.3	18 22.8	+31.8	6.4	83
84	11 56.5	+31.0	5.3	12 56.2	+31.2	5.3	13 56.0	+31.2	5.4	14 55.7	+31.3	5.4	15 55.5	+31.3	5.4	16 55.2	+31.4	5.4	17 54.9	+31.5	5.4	18 54.6	+31.6	5.5	84
85	12 27.5	+30.9	4.4	13 27.4	+30.9	4.5	14 27.2	+31.0	4.5	15 27.0	+31.1	4.5	16 26.8	+31.1	4.5	17 26.6	+31.2	4.5	18 26.4	+31.3	4.6	19 26.2	+31.4	4.6	85
86	12 58.4	+30.7	3.6	13 58.3	+30.7	3.6	14 58.2	+30.8	3.6	15 58.1	+30.8	3.6	16 57.9	+30.9	3.6	17 57.8	+31.0	3.6	18 57.7	+31.0	3.7	19 57.6	+31.0	3.7	86
87	13 29.1	+30.5	2.7	14 29.0	+30.6	2.7	15 29.0	+30.5	2.7	16 28.9	+30.6	2.7	17 28.8	+30.7	2.7	18 28.8	+30.6	2.7	19 28.7	+30.7	2.8	20 28.6	+30.8	2.8	87
88	13 59.6	+30.3	1.8	14 59.6	+30.3	1.8	15 59.5	+30.4	1.8	16 59.5	+30.4	1.8	17 59.5	+30.4	1.8	18 59.4	+30.5	1.8	19 59.4	+30.5	1.8	20 59.4	+30.4	1.9	88
89	14 29.9	+30.1	0.9	15 29.9	+30.1	0.9	16 29.9	+30.1	0.9	17 29.9	+30.1	0.9	18 29.9	+30.1	0.9	19 29.9	+30.1	0.9	20 29.9	+30.1	0.9	21 29.8	+30.2	0.9	89
90	15 00.0	+29.9	0.0	16 00.0	+29.9	0.0	17 00.0	+29.9	0.0	18 00.0	+29.9	0.0	19 00.0	+29.9	0.0	20 00.0	+29.9	0.0	21 00.0	+29.8	0.0	22 00.0	+29.8	0.0	90
	15°			16°			17°			18°			19°			20°			21°			22°			

Central diagram labels: Z(N), Q, Z 103°.9, Dec. 5° S, LHA 60°, Pn, Lat. 15° N, NORTH DECLINATION, CELESTIAL HORIZON, N, S, Alt. 27° 17'.7, SOUTH DECLINATION, Ps, t 120° E, CELESTIAL EQUATOR, PRIME VERTICAL, Z 76°.1, Q', Z(S).

Figures 1 and 2 illustrate four of the eight possible celestial triangles for specific numerical values of latitude and declination and the LHA's tabulated on the left and right-hand pages of an opening of the tables.

The diagram on the plane of the celestial meridian in figure 1 indicates that the celestial body always lies above the celestial horizon when the observer's latitude has the same name as the declination of the body and the values of LHA are those tabulated on the left-hand page of an opening of the tables. The diagram in figure 2 reveals that for the various combinations of arguments on the right-hand page, including whether the name of the observer's latitude is the same as or contrary to the name of the declination, the numerical value of the declination governs whether the body is above or below the celestial horizon. For example, the following arguments are used for entering the tables:

LHA	60°	
Latitude	15° N	(Contrary Name to Declination)
Declination	5° S	

The respondents are:

Tabular altitude,	ht	(Tab. Hc)	27°17.7′
Altitude difference,	d		(−)20.5′
Azimuth angle,	Z		103.9°

As can be verified by an inspection of figures 2 and 4a, the altitude respondent is for a body 27°17.7′ above the celestial horizon. Further inspection of these figures reveals that with the LHA and latitude (Contrary Name) remaining constant, the altitude of the body decreases as the declination increases. Between values of declination 61° and 62° the body crosses the celestial horizon. When the declination reaches 70°, the altitude is 4°28.5′ below the celestial horizon; the tabular azimuth angle is the supplement of the actual azimuth angle of 162.7°.

As an additional example, the following arguments are used for entering the tables:

LHA	240°	(t 120°E)
Latitude	15° S	(Same Name as Declination)
Declination	5° S	

The respondents are:

Tabular altitude,	ht	(Tab. Hc)	27°17.7′
Altitude difference,	d		(−)20.5′
Azimuth angle,	Z		103.9°

However, inspection of the diagram on the plane of the celestial meridian in figures 2 and 4b reveals that the altitude is 27°17.7′ *below* the celestial horizon; the tabular azimuth angle is the *supplement* of the actual azimuth angle of 76.1°. Further inspection of these figures reveals that with the LHA and latitude (Same Name) remaining constant, the altitude of the body increases as the declination increases. Between values of declination of 61° and 62° the body crosses the celestial horizon. When the declination reaches 70°, the altitude is 4°28.5′ above the celestial horizon; the tabular azimuth angle is the actual azimuth angle of 17.3°.

Inspection of figures 1, 2, and 3 reveals that if the left-hand page of an opening of the tables is entered with latitude of contrary name and one of the LHA's tabulated at the bottom of the facing page, the tabular altitudes are negative; the tabular azimuth angles are the supplements of the actual azimuth angles.

A. DESCRIPTION OF TABLES

$Z_{(N)}$, zenith of observer at latitude 15° N.
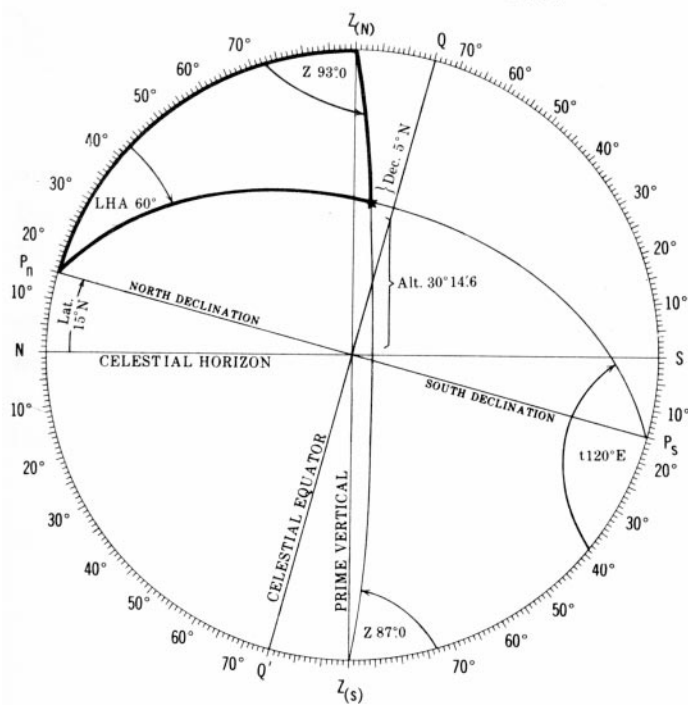
FIGURE 3a
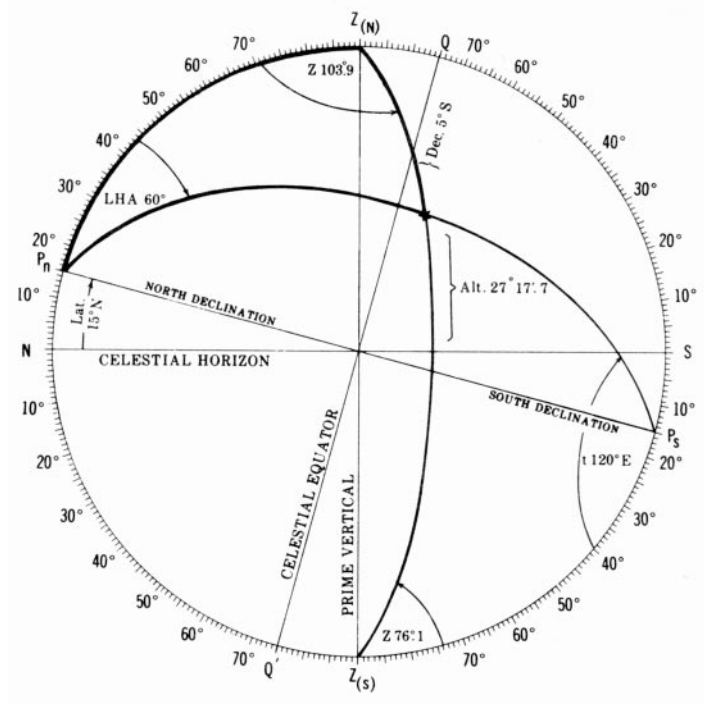
FIGURE 4a

$Z_{(S)}$, zenith of observer at latitude 15° S.
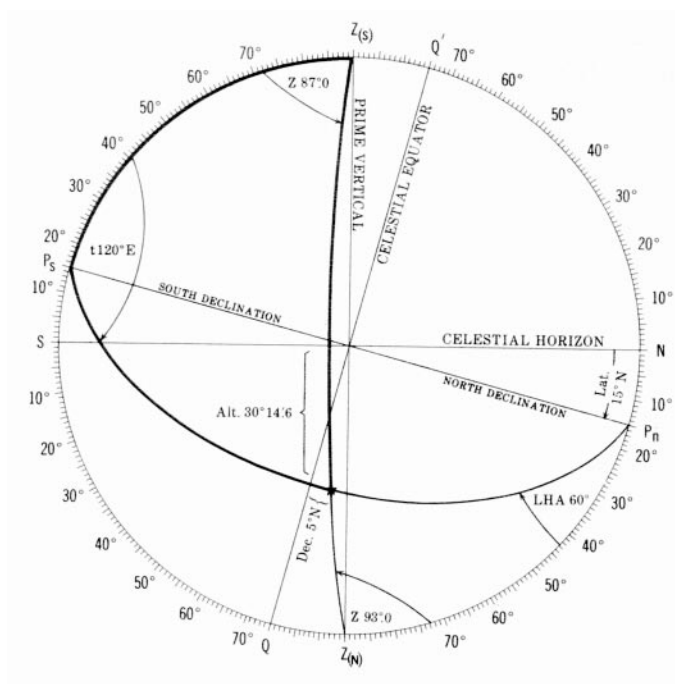
FIGURE 3b
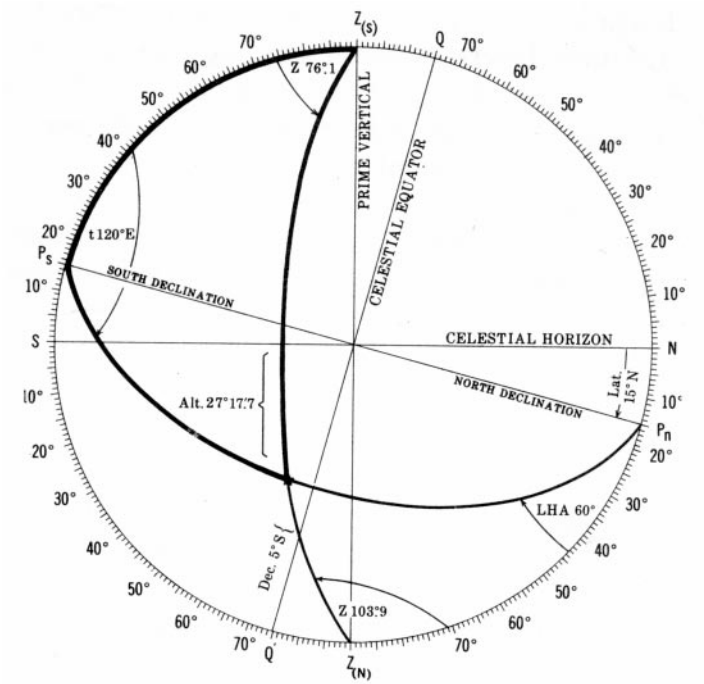
FIGURE 4b

B. INTERPOLATION

1. Requirements. In the normal use of the tables with the Marcq Saint Hilaire method, it is only necessary to interpolate the tabular altitude and azimuth angle for the excess of the actual declination of the celestial body over the integral declination argument. When the tabular altitude is less than 60°, the required interpolation can always be effected through the use of the tabulated altitude differences. When the tabular altitude is in excess of 60°, it may be necessary to include the effects of second differences. When the tabular altitude difference is printed in italic type followed by a small dot, the effects of the second differences should be included in the interpolation. Although the effects of second differences may not be required, these effects can always be included in the interpolation whenever it is desired to obtain greater accuracy.

If the sight reduction is from a position such that interpolation for latitude and local hour angle increments is necessary, the required additional interpolation of the altitude can be effected by graphical means.

2. First and Second Differences. The data in the column for latitude 15° (Same Name as Declination) as contained in figure 1 is rearranged in Table I to illustrate the first and second differences.

TABLE I

LHA 60°, Lat. 15° (Same Name as Declination)

Dec.	ht (Tab. Hc)	First Difference	Second Difference
4°	29°59.4′		
		+15.2′	
5°	30°14.6′		-0.5′
		+14.7′	
6°	30°29.3′		-0.6′
		+14.1′	
7°	30°43.4′		

Table I illustrates that the first differences are the differences between successive altitudes in a latitude column; the second differences are the differences between successive first differences.

3. Linear Interpolation. The usual case is that the change of altitude with 60′ increase in declination is nearly linear as illustrated in figure 5. In this case, the required interpolation can be effected by multiplying the altitude difference (a first difference) by the excess of the actual declination over the integral declination argument divided by 60′. This excess of declination in minutes and tenths of minutes of arc is referred to as the declination increment and is abbreviated Dec. Inc.

Using the data of Table I, the computed altitude when the LHA is 60°, the latitude (Same Name) is 15°, and the declination is 5°45.5′ is determined as follows:
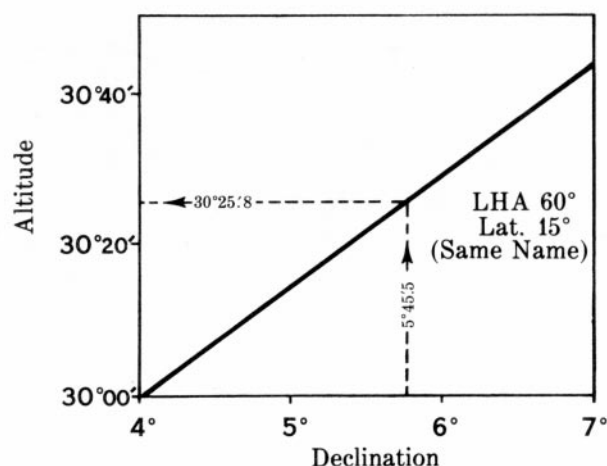
$$\text{Correction} = \text{Altitude difference} \times \frac{\text{Dec. Inc.}}{60'} = (+)14.7' \times \frac{45.5'}{60'} = 11.2'$$

$$Hc = ht + correction = 30°14.6' + 11.2' = 30°25.8'$$

FIGURE 5

B. INTERPOLATION

4. The Interpolation Table.

(a) Design. The main part of the four-page Interpolation Table is basically a multiplication table providing tabulations of:

$$\text{Altitude Difference} \times \frac{\text{Declination Increment}}{60'}$$

The design of the Interpolation Table is such that the desired product must be derived from component parts of the altitude difference. The first part is a multiple of 10′ (10′, 20′, 30′, 40′, or 50′) of the altitude difference; the second part is the remainder in the range 0.0′ to 9.9′. For example, the component parts of altitude difference 14.7′ are 10′ and 4.7′.

In the use of the first part of the altitude difference, the Interpolation Table arguments are Dec. Inc. and the integral multiple of 10′ in the altitude difference, d. The respondent is:

$$\text{Tens} \times \frac{\text{Dec. Inc.}}{60'} \text{ (See figure 6)}$$

In the use of the second part of the altitude difference, the Interpolation Table arguments are the nearest Dec. Inc. ending in 0.5′ and Units and Decimals. The respondent is:

$$\text{Units and Decimals} \times \frac{\text{Dec. Inc.}}{60'}$$

FIGURE 6

In computing the table, the values in the Tens part of the multiplication table were modified by small quantities varying from −0.042′ to +0.033′ before rounding to the tabular precision to compensate for any difference between the actual Dec. Inc. and the nearest Dec. Inc. ending in 0.5′ when using the Units and Decimals part of the table.

(b) Instructions for use of the Interpolation Table.

    (i)   Turn to the Interpolation Table on the inside front cover and facing page if the Dec. Inc. is in the range 0.0′ to 31.9′ or on the inside back cover and facing page if the Dec. Inc. is in the range 28.0′ to 59.9′.

    (ii)   Enter the Interpolation Table with Dec. Inc. as the vertical argument.

    (iii)   On the same horizontal line as the Dec. Inc., extract the altitude correction for the first part of the altitude difference from the appropriate Tens column.

    (iv)   From the Units and Decimals subtable immediately to the right, extract the altitude correction for the second part of the altitude difference.

    (v)   Add the two parts to form the correction to the tabular altitude for declination increment. The sign of the correction is in accordance with the sign of the altitude difference, d.

    (vi)   When the altitude difference, d, is printed in italic type followed by a small dot, enter that compartment of the DSD table opposite the block in which the Dec. Inc. is found with the DSD as the argument to obtain the DSD correction to the altitude. The DSD correction is always plus. (See section B.5)

(c) Example of the Use of Interpolation Table. As an example of the use of the Interpolation Table, the computed altitude and true azimuth are determined for Lat. 15°N, LHA 60°, and Dec. 5°45.5′ N. Data are exhibited in figure 7.

The respondents for the entering arguments (Lat. 15° Same Name as Declination, LHA 60°, and Dec. 5°) are:

tabular altitude,	ht	30°14.6′
altitude difference,	d	(+)14.7′
tabular azimuth angle,	Z	93.0°

Note that Dec. Inc. 45.5′ is the vertical argument for entering the Interpolation Table to extract the correction for tens of minutes of altitude difference, d, and that it also indicates the subtable where the correction for minutes and tenths of minutes (Units and Decimals) of altitude difference, d, is found. Entering the Interpolation Table with Dec. Inc. 45.5′ as the vertical argument, the correction for 10′ of the altitude difference is 7.6′; the correction for 4.7′ of the altitude difference is 3.6′. Adding the two parts, the correction is (+)11.2′, the sign of the correction being in accordance with the sign of the altitude difference, d.

No special table is provided for interpolation of the azimuth angle, and the differences are not tabulated. With latitude and local hour angle constant, the successive azimuth angle differences corresponding to 1° increase in declination are less than 10.0° for altitudes less than 84°, and can easily be found by inspection. If formal interpolation of azimuth angle is desired, the degrees and tenths of degrees of azimuth angle difference are treated as minutes and tenths of minutes in obtaining the required correction from the Units and Decimals subtable to the right of the declination increment. But for most practical applications, interpolation by inspection usually suffices. In this example of formal interpolation, using an azimuth angle difference of −1.2° and a Dec. Inc. of 45.5′, the correction as extracted from the Units and Decimals subtable to the right of the Dec. Inc. is −0.9°. Therefore, the azimuth angle as interpolated for declination increment is 92.1° (93.0° −0.9°). In summary,

tabular altitude	ht	30° 14.6′	tabular azimuth angle	Z	93.0°
correction for 10′ of alt. diff.		(+) 7.6′	correction for Dec. Inc. 45.5′		(−)0.9°
correction for 4.7′ of alt. diff.		(+) 3.6′			
computed altitude	Hc	30° 25.8′	interpolated azimuth angle	Z	N92.1°W
		(See figures 5 and 7)	true azimuth	Zn	267.9°

Data from Page 122

Data from Interpolation Table

FIGURE 7

5. Interpolation when Second Differences are Required. The accuracy of linear interpolation usually decreases as the altitude increases. At altitudes above 60° it may be necessary to include the effect of second differences in the interpolation. When the altitude difference, d, is printed in italic type followed by a small dot, the second-difference correction may exceed 0.25′, and should normally be applied. The need for a second-difference correction is illustrated by the graph of Table II data in figure 8.

TABLE II

LHA 28°, Lat. 15° (Same Name as Declination)

Dec.	ht (Tab. Hc)	First Difference	Second Difference
15°	62°58.4′		
		+2.8′·	
16°	63°01.2′		−2.0′
		+0.8′·	
17°	63°02.0′		−2.1′
		−1.3′·	
18°	63°00.7′		
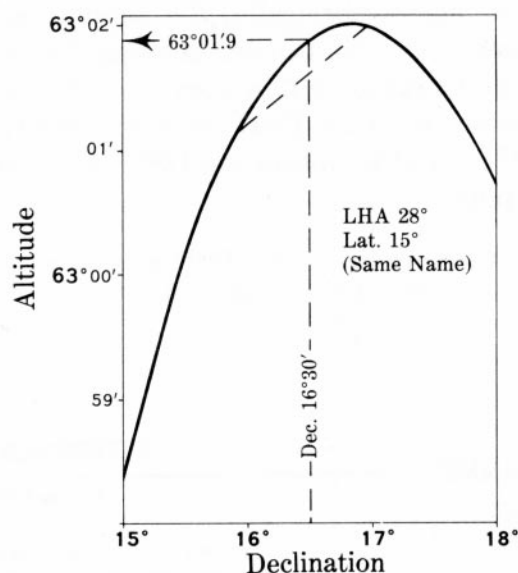
FIGURE 8

Other than graphically, the required correction for the effects of second differences is obtained from the appropriate subtable of the Interpolation Table. However, before the Interpolation Table can be used for this purpose, what is known as the double-second difference (DSD) must be formed.

(a) Forming the Double-Second Difference (DSD)

The double-second difference is the sum of two successive second differences. Although second differences are not tabulated, the DSD can be formed readily by subtracting, algebraically, the tabular altitude difference immediately above the respondent altitude difference from the tabular altitude difference immediately below. The result will always be a negative value.

(b) The Double-Second Difference Correction

As shown in figure 9, that compartment of the DSD table opposite the block in which the Dec. Inc. is found is entered with the DSD to obtain the DSD correction to the altitude. The correction is always plus. Therefore, the sign of the DSD need not be recorded. When the DSD entry corresponds to an exact tabular value, always use the upper of the two possible corrections.

(c) ***Example of the Use of the Double-Second Difference.***

As an example of the use of the double-second difference (DSD) the computed altitude and true azimuth are determined for Lat. 15°N, LHA 28°, and Dec. 16°30.0′N. Data are exhibited in figure 9.

The respondents for the entering arguments (Lat. 15° Same Name as Declination, LHA 28°, and Dec. 16°) are:

tabular altitude,	ht	63°01.2′
altitude difference,	d	(+)0.8′.
azimuth angle,	Z	84.1°

The linear interpolation correction to the tabular altitude for Dec. Inc. 30.0′ is (+)0.4′.

$$Hc = ht + \text{linear correction} = 63°01.2′ + 0.4′ = 63°01.6′$$

However, by inspection of figure 8, illustrating this solution graphically, the computed altitude should be 63°01.9′. The actual change in altitude with an increase in declination is nonlinear. The altitude value lies on the curve between the points for declination 16° and declination 17° instead of the straight line connecting these points.

The DSD is formed by subtracting, algebraically, the tabular altitude difference immediately above the respondent altitude difference from the tabular altitude difference immediately below. Thus, the DSD is formed by algebraically subtracting (+)2.8′ from (−)1.3′; the result is (−)4.1′.

As shown in figure 9, that compartment of the DSD table opposite the block in which the Dec. Inc. (30.0′) is found is entered with the DSD (4.1′) to obtain the DSD correction to the altitude. The correction is 0.3′. The correction is always plus.

$$Hc = ht + \text{linear correction} + \text{DSD correction}$$
$$Hc = 63°01.2′ + 0.4′ + 0.3′ = 63°01.9′$$

28°, 332° L.H.A.

LATITUDE SAME NAME

Dec.	15° Hc	15° d	15° Z	16° Hc	16° d	16° Z	17° Hc	17° d	17° Z
°	° ′	′	°	° ′	′	°	° ′	′	°
9	61 59.3	+ 14.7	99.1	61 48.8	+ 16.8	101.0	61 36.4	+ 18.9	102.8
10	62 14.0	+ 12.8	97.1	62 05.6	+ 15.0	98.9	61 55.3	+ 17.1	100.8
11	62 26.8	+ 10.9	95.0	62 20.6	+ 13.1	96.9	62 12.4	+ 15.3	98.8
12	62 37.7	+ 8.9 •	92.8	62 33.7	+ 11.1	94.8	62 27.7	+ 13.4	96.7
13	62 46.6	+ 6.9 •	90.7	62 44.8	+ 9.2	92.6	62 41.1	+ 11.4	94.6
14	62 53.5	+ 4.9 •	88.5	62 54.0	+ 7.2 •	90.5	62 52.5	+ 9.5 •	92.4
15	62 58.4	⌈+ 2.8 • ⌉	86.3	63 01.2	+ 5.1 •	88.3	63 02.0	+ 7.4 •	90.2
16	63 01.2	+ 0.8 •	84.1	63 06.3	+ 3.1 •	86.1	63 09.4	+ 5.4 •	88.0
17	63 02.0	∟− 1.3 • ⌐81.9		63 09.4	+ 1.0 •	83.9	63 14.8	+ 3.3 •	85.8
18	63 00.7	− 3.3 •	79.7	63 10.4	− 1.0 •	81.6	63 18.1	+ 1.3 •	83.6
19	62 57.4	− 5.4 •	77.5	63 09.4	− 3.1 •	79.4	63 19.4	− 0.9 •	81.4

Data from Page 58

INTERPOLATION TABLE

Dec. Inc.	Altitude Difference (d) Tens 10′	20′	30′	40′	50′	Decimals ⇓	Units 0′	1′	2′	3′	4′	5′	6′	7′	8′	9′	Double Second Diff. and Corr.
′	′	′	′	′	′		′	′	′	′	′	′	′	′	′	′	′ ′
30.0	5.0	10.0	15.0	20.0	25.0	.0	0.0	0.5	1.0	1.5	2.0	2.5	3.0	3.6	4.1	4.6	0.8 0.1
30.1	5.0	10.0	15.0	20.0	25.1	.1	0.1	0.6	1.1	1.6	2.1	2.6	3.1	3.6	4.1	4.6	2.4 0.2
30.2	5.0	10.0	15.1	20.1	25.1	.2	0.1	0.6	1.1	1.6	2.1	2.6	3.2	3.7	4.2	4.7	4.0
30.3	5.0	10.1	15.1	20.2	25.2	.3	0.2	0.7	1.2	1.7	2.2	2.7	3.2	3.7	4.2	4.7	5.6 0.3
30.4	5.1	10.1	15.2	20.3	25.3	.4	0.2	0.7	1.2	1.7	2.2	2.7	3.3	3.8	4.3	4.8	7.2 0.4
30.5	5.1	10.2	15.3	20.3	25.4	.5	0.3	0.8	1.3	1.8	2.3	2.8	3.3	3.8	4.3	4.8	8.8 0.5
30.6	5.1	10.2	15.3	20.4	25.5	.6	0.3	0.8	1.3	1.8	2.3	2.8	3.4	3.9	4.4	4.9	10.4 0.6
30.7	5.1	10.3	15.4	20.5	25.6	.7	0.4	0.9	1.4	1.9	2.4	2.9	3.4	3.9	4.4	4.9	12.0 0.7
30.8	5.2	10.3	15.4	20.6	25.7	►.8	0.4	0.9	1.4	1.9	2.4	2.9	3.5	4.0	4.5	5.0	13.6 0.8
30.9	5.2	10.3	15.5	20.6	25.8	.9	0.5	1.0	1.5	2.0	2.5	3.0	3.5	4.0	4.5	5.0	15.2 0.9
																	16.8 1.0

Data from Interpolation Table

FIGURE 9

C. SPECIAL TECHNIQUES

1. Adjustment of Straight Line of Position. The Table of Offsets gives the corrections to the straight line of position (LOP) as drawn on a chart or plotting sheet to provide a closer approximation to the arc of the circle of equal altitude, a small circle of radius equal to the zenith distance. As shown in figure 10, the corrections are offsets of points on the LOP and are drawn at right angles to the LOP in the direction of the observed body. The offset points are joined to obtain the arc of the small circle. Usually the desired approximation to the arc of the small circle can be obtained by drawing a straight line through two offset points. The magnitudes of the offsets are dependent upon altitude and the distance of the offset point from the intercept.
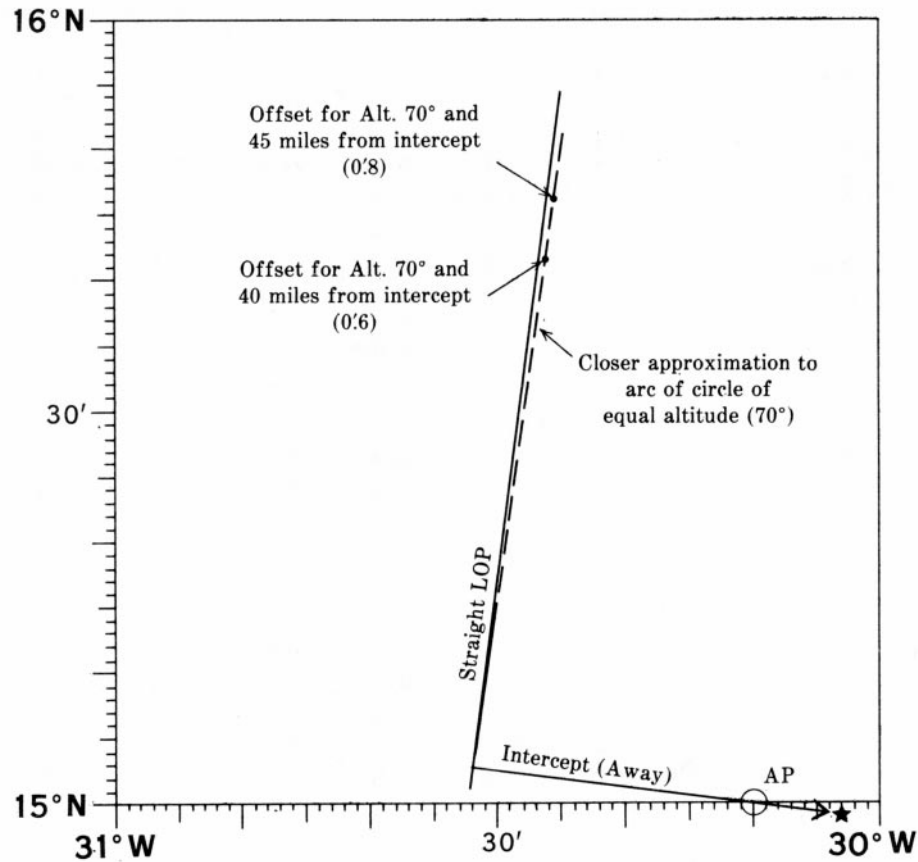
FIGURE 10

TABLE OF OFFSETS

DISTANCE ALONG LINE OF POSITION FROM INTERCEPT

ALT.	00′	05′	10′	15′	20′	25′	30′	35′	40′	45′	ALT.
					OFFSETS						
0°	0.0′	0.0′	0.0′	0.0′	0.0′	0.0′	0.0′	0.0′	0.0′	0.0′	0°
30	0.0	0.0	0.0	0.0	0.0	0.1	0.1	0.1	0.1	0.2	30
40	0.0	0.0	0.0	0.0	0.1	0.1	0.1	0.2	0.2	0.3	40
50	0.0	0.0	0.0	0.0	0.1	0.1	0.2	0.2	0.3	0.3	50
55	0.0	0.0	0.0	0.0	0.1	0.1	0.2	0.3	0.3	0.4	55
60	0.0	0.0	0.0	0.1	0.1	0.2	0.2	0.3	0.4	0.5	60
62	0.0	0.0	0.0	0.1	0.1	0.2	0.2	0.3	0.4	0.5	62
64	0.0	0.0	0.0	0.1	0.1	0.2	0.3	0.4	0.5	0.6	64
66	0.0	0.0	0.0	0.1	0.1	0.2	0.3	0.4	0.5	0.7	66
68	0.0	0.0	0.0	0.1	0.1	0.2	0.3	0.4	0.6	0.7	68
70	0.0	0.0	0.0	0.1	0.2	0.2	0.4	0.5	0.6	0.8	70
71	0.0	0.0	0.0	0.1	0.2	0.3	0.4	0.5	0.7	0.9	71
72	0.0	0.0	0.0	0.1	0.2	0.3	0.4	0.5	0.7	0.9	72
73	0.0	0.0	0.0	0.1	0.2	0.3	0.4	0.6	0.8	1.0	73
74	0.0	0.0	0.1	0.1	0.2	0.3	0.5	0.6	0.8	1.0	74
75	0.0	0.0	0.1	0.1	0.2	0.3	0.5	0.7	0.9	1.1	75
76	0.0	0.0	0.1	0.1	0.2	0.4	0.5	0.7	0.9	1.2	76
77	0.0	0.0	0.1	0.1	0.3	0.4	0.6	0.8	1.0	1.3	77
78	0.0	0.0	0.1	0.2	0.3	0.4	0.6	0.8	1.1	1.4	78
79	0.0	0.0	0.1	0.2	0.3	0.5	0.7	0.9	1.2	1.5	79
80.0	0.0	0.0	0.1	0.2	0.3	0.5	0.7	1.0	1.3	1.7	80.0
80.5	0.0	0.0	0.1	0.2	0.3	0.5	0.8	1.1	1.4	1.8	80.5
81.0	0.0	0.0	0.1	0.2	0.4	0.6	0.8	1.1	1.5	1.9	81.0
81.5	0.0	0.0	0.1	0.2	0.4	0.6	0.9	1.2	1.6	2.0	81.5
82.0	0.0	0.0	0.1	0.2	0.4	0.6	0.9	1.3	1.7	2.1	82.0
82.5	0.0	0.0	0.1	0.2	0.4	0.7	1.0	1.4	1.8	2.2	82.5
83.0	0.0	0.0	0.1	0.3	0.5	0.7	1.1	1.5	1.9	2.4	83.0
83.5	0.0	0.0	0.1	0.3	0.5	0.8	1.2	1.6	2.0	2.6	83.5
84.0	0.0	0.0	0.1	0.3	0.5	0.9	1.2	1.7	2.2	2.8	84.0
84.5	0.0	0.0	0.2	0.3	0.6	1.0	1.4	1.9	2.4	3.1	84.5
85.0	0.0	0.0	0.2	0.4	0.7	1.0	1.5	2.1	2.7	3.4	85.0
85.5	0.0	0.0	0.2	0.4	0.7	1.2	1.7	2.3	3.0	3.8	85.5
86.0	0.0	0.1	0.2	0.5	0.8	1.3	1.9	2.6	3.4	4.3	86.0
86.5	0.0	0.1	0.2	0.5	1.0	1.5	2.2	2.9	3.8	4.9	86.5
87.0	0.0	0.1	0.3	0.6	1.1	1.7	2.5	3.4	4.5	5.7	87.0
87.5	0.0	0.1	0.3	0.8	1.3	2.1	3.0	4.1	5.4	6.9	87.5
88.0	0.0	0.1	0.4	0.9	1.7	2.7	3.8	5.2	6.9	8.8	88.0
88.5	0.0	0.2	0.6	1.3	2.3	3.5	5.1	7.1	9.4	12.1	88.5
89.0	0.0	0.3	0.8	1.9	3.4	5.5	8.0	11.3	15.3	20.3	89.0

In adjusting the straight LOP to obtain a closer approximation to the arc of the circle of equal altitude, points on the LOP are offset at right angles to the LOP in the direction of the celestial body. The arguments for entering the table are the distance from the intercept to the point on the LOP to be offset and the altitude of the body.

In the use of the table with the graphical method for interpolating altitude for latitude and LHA increments, the offset of the foot of the perpendicular is along the azimuth line in a direction away from the body. The arguments for entering the table are the distance from the DR to the foot of the perpendicular and the altitude of the body.

2. Interpolation for Latitude and Local Hour Angle. The following graphical method can be used to interpolate the altitude for latitude and local hour angle increments. *The basic method should have most frequent application in great-circle solutions.*

In principle the method is the measurement of the difference of the radii of two circles of equal altitude corresponding to the altitudes of a celestial body from two positions at the same instant. One circle passes through the assumed position (AP), and the second circle passes through the dead reckoning position (DR) or other position from which the computed altitude is required.

The measurement, which is the difference in zenith distances as measured from the zenith of the assumed position and the zenith of some nearby position, is effected as follows:

(1) Draw the azimuth line from the assumed position (AP) as shown in figure 11 (the azimuth angle is interpolated for declination increment before conversion to true azimuth).

(2) From the position (DR) for which the computed altitude is required, draw a line perpendicular to the azimuth line or its extension. This line approximates the arc of the circle of equal altitude passing through the DR.

(3) Measure the distance from the foot of the perpendicular to the DR in nautical miles.

(4) Entering the Table of Offsets with the distance of the DR from the foot of the perpendicular and the altitude of the body as interpolated for declination increment, extract the offset.

(5) From the foot of the perpendicular and in a direction away from the celestial body, lay off the offset on the azimuth line or its extension.

(6) As shown in figure 11, a closer approximation to the arc of the circle of equal altitude through the DR is made by drawing a straight line from the offset point to the DR.

(7) The required correction, in units of minutes of latitude, for the latitude and LHA increments is the length along the azimuth line between the AP and the arc of the circle of equal altitude through the DR.

If the arc of the circle of equal altitude through the DR crosses the azimuth line between the AP and the body, the correction is to be added to the altitude interpolated for declination increment; otherwise the correction is to be subtracted. The method will give highly satisfactory results except when plotting on a Mercator chart in high latitudes.
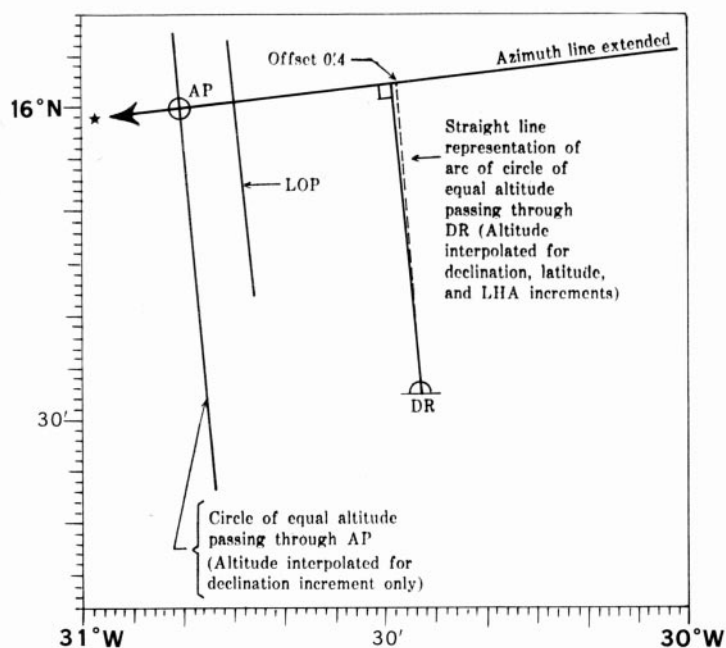
Example:

Computed altitude from AP	Hc	70°05.0′
Observed altitude	Ho	70°00.0′
Intercept	a	5.0 A

Computed altitude from AP	Hc	70°05.0′
Difference of the radii		20.4′
Computed altitude from DR	Hc	69°44.6′

Computed altitude from DR	Hc	69°44.6′
Observed altitude	Ho	70°00.0′
Intercept	a	15.4 T

FIGURE 11

3. Interpolation near the Horizon. This discussion is restricted to the interpolation of altitude for declination within the 1° interval containing the horizon, indicated by the horizontal segments of the C-S Line. Interpolation of altitude in the interval under consideration is accomplished by using the last tabular altitude and altitude difference appearing above the C-S Line. Since the last tabular altitude above the C-S Line indicates the body's altitude above the horizon for LHA at top of page, for the pertinent latitude, and for the last integral declination above the horizontal segment of the C-S Line pertaining to that particular latitude, interpolation

resulting in positive altitudes may be carried out for increments of declination of contrary name so long as the interpolated altitude correction does not exceed the last tabular altitude above the C-S Line; for the LHA at bottom of page, positive altitudes will result when interpolating altitude for increments of declination of same name so long as the interpolated altitude correction exceeds the last tabular value above the C-S Line. Interpolation for declinations and increments of declination in excess of the above limits results in negative altitudes.

The tabular azimuth angle pertinent to this one-degree interval of declination is that immediately above or that immediately below the C-S Line, according as the entering arguments are contrary or same name, respectively. The difference in azimuth angle for the interval is determined by taking the value of tabular azimuth angle, on the same side of the C-S Line as the LHA argument, from the supplement of that on the opposite side of the line.

4. Negative Altitudes. This paragraph is restricted to tabular and interpolated altitudes for declinations other than one-degree intervals of declination containing the C-S Line. For all local hour angles at the top of the right-hand page, all tabular or interpolated altitudes on that page for declinations below the C-S Line are negative; also for any local hour angle at the bottom of the right-hand page, all tabular or interpolated altitudes for declinations above the C-S Line are negative; additionally, for these same local hour angles and latitudes changed to Contrary Name, the tabular or interpolated altitudes on the left-hand page are negative. Interpolation of altitudes for declination increments within these areas of negative altitude should, however, be accomplished as if the altitudes were positive, adhering strictly to the sign given to d. Then, after interpolation, regard the results as negative. In all instances involving negative altitudes, except the one-degree interval of declination which includes the C-S Line, the supplement of the pertinent tabular azimuth angle is that to be converted to true azimuth by the rules to be found on each opening of the basic tables.

5. Interpolation near the Zenith. In the region within 4° of the zenith where normal interpolation methods are inadequate, the following method can usually be used to interpolate both altitude and azimuth angle. The Interpolation Table is employed in carrying out the desired interpolation, but the values of altitude and azimuth angle extracted from the basic tables constitute data which require independent differencing; the tabular altitude difference, d, is not used.

To carry out the altitude interpolation, the basic tables are entered with the pertinent LHA and Dec., and with the integral degree of Lat. so chosen that, when increased by the declination increment, it is within 30′ of the known or DR latitude; this practice will prevent long intercepts. For these entering arguments and for a latitude and declination one degree more than the above referenced latitude and declination, respectively, extract the tabular altitudes and azimuth angles. The altitudes and azimuth angles are then differenced and with these differences interpolation of altitude and azimuth angle for the desired declination is made, utilizing the Interpolation Table. The computed altitude is then compared with that observed to determine the intercept, which together with the interpolated azimuth angle converted to true azimuth makes possible the construction of a line of position, which is plotted from the assumed longitude, and from the latitude of the entering argument, augmented by the declination increment.

Example	LHA	Lat.	Dec.	Ho
i	3°18′	24°12′S	21°33.3′S	85°58.2′
ii	356°52′	28°14′S	30°19.7′S	86°33.4′

Example i

Lat.	Dec.	Tab. Hc	diff.	Tab. Z	diff.
24°	21°	85°55.0′		136.7°	
			(+)0.8′		(+)0.2°
25°	22°	85°55.8′		136.9°	

Interpolate to Dec.=21°33.3′
Dec. Inc.=33.3′, diff.=(+)0.8′, Z diff.=(+)0.2°

Tab. Hc	85°55.0′	Tab. Z	136.7°
Correction	(+) 0.4		(+) 0.1
Hc	85°55.4′	Z	136.8°
Ho	85°58.2′		
Intercept	2.8 T	Zn	316.8°

Plot from Lat. 24°33.3′ S

Example ii

Lat.	Dec.	Tab. Hc	diff.	Tab. Z	diff
28°	30°	86°42.1′		52.0°	
			(+)1.2′		(-)0.3°
29°	31°	86°43.3′		51.7°	

Interpolate to Dec.=30°19.7′
Dec. Inc.=19.7′, diff.=(+)1.2′, Z diff.=(-)0.3°

Tab. Hc	86°42.1′	Tab. Z	52.0°
Correction	(+) 0.4		(-) 0.1
Hc	86°42.5′	Z	51.9°
Ho	86°33.4′		
Intercept	9.1 A	Zn	128.1°

Plot from Lat. 28°19.7′ S

D. OTHER APPLICATIONS

1. Star Identification. Although no formal star identification tables are included in these volumes, a simple approach to star identification is to scan the pages of the appropriate latitudes and observe the combination of arguments which give the altitude and azimuth angle of the observation. Thus the declination and LHA☆ are determined directly. The star's SHA is found from, SHA☆ = LHA☆−LHAϓ. From these quantities the star can be identified from *The Nautical Almanac.*

Another solution is available through an interchange of arguments using the nearest integral values. The procedure consists of entering the tables with the observer's latitude (Same Name as Declination), with the observed azimuth angle (converted from observed true azimuth as required) as LHA and the observed altitude as declination, and extracting from the tables the altitude and azimuth angle respondents. The extracted altitude becomes the body's declination; the extracted azimuth angle (or its supplement) is the meridian angle of the body. Note that the tables are always entered with latitude of same name as declination. In north latitudes the tables can be entered with true azimuth as LHA.

If the respondents are extracted from above the C-S Line on a right-hand page, the name of the latitude is actually contrary to that of the declination. Otherwise, the declination of the body has the same name as the latitude. If the azimuth angle respondent is extracted from above the C-S Line, the supplement of the tabular value is the meridian angle, t, of the body. If the body is east of the observer's meridian, LHA = 360° − t; if the body is west of the meridian, LHA = t.

EXAMPLES FOR STAR IDENTIFICATION (Selection for illustration only)

Ex.	Lat.	Long.	Obs. Alt.	Obs. Zn	LHAϓ*
1	17° 15′ N	33° 55′ W	54° 36′	20°	189°
2	15 06 N	143 40 W	40 00	96	64
3	15 54 N	168 10 E	19 22	131	288
4	20 38 N	27 27 W	56 56	260	185
5	16 22 N	66 42 E	50 17	235	110
6	18 43 N	165 19 W	47 25	317	351
7	20 55 S	77 33 E	39 32	87	149
8	15 28 S	60 14 E	43 46	22	97
9	19 12 S	34 02 W	45 22	156	274
10	27 43 S	49 17 E	19 35	220	190
11	23 04 S	24 22 W	60 57	276	214
12	15 24 S	127 14 E	31 08	337	305

*LHAϓ from *The Nautical Almanac* for date and GMT of observation.

SOLUTIONS

	Entering Argument					Star Coordinates and Identity				
Ex.	Lat	LHA	Dec.	Page		Dec.	t	LHA☆	SHA☆	Name
1	17°	20°	55°	Left		49°N	17° E	343°	154°	*Alkaid*
2	15	96	40	Right, below C-S Line		5 N	50 E	310	246	*Procyon*
3	16	131	19	Right, above C-S Line		30 S	56 E	304	16	*Fomalhaut*
4	21	260	57	Right, below C-S Line		12 N	33 W	33	208	*Regulus*
5	16	235	50	Right, above C-S Line		8 S	32 W	32	282	*Rigel*
6	19	317	47	Left		45 N	41 W	41	50	*Deneb*
7	21	180-87=93	40	Right, below C-S Line		11 S	51 E	309	160	*Spica*
8	15	180-22=158	44	Right, above C-S Line		28 N	18 E	342	245	*Pollux*
9	19	180-156=24	45	Left		57 S	32 E	328	54	*Peacock*
10	28	220-180=40	20	Left		53 S	93 W	93	263	*Canopus*
11	23	276-180=96	61	Right, below C-S Line		17 S	30 W	30	176	*Gienah*
12	15	337-180=157	31	Right, above C-S Line		39 N	25 W	25	80	*Vega*

SHA☆ = LHA☆−LHAϓ

2. Great-Circle Sailing. The great-circle distance between any two points on the assumed spherical surface of the Earth and the initial great-circle course angle may be found by relating the problems to the solution of the celestial triangle. For by entering the tables with latitude of departure as latitude, latitude of destination as declination, and difference of longitude as LHA, the tabular altitude and azimuth angle may be extracted and converted to distance and course.

The tabular azimuth angle (or its supplement) becomes the initial great-circle course angle, prefixed N or S for the latitude of departure, and suffixed E or W depending upon the destination being east or west of point of departure.

If all entering arguments are integral degrees, the altitude and azimuth angle are obtained directly from the tables without interpolation. If the latitude of destination is nonintegral, interpolation for the additional minutes of latitude is done as in correcting altitude for any declination increment; if either the latitude of departure or difference of longitude, or both, are nonintegral, the additional interpolation is done graphically.

Since the latitude of destination becomes the declination entry, and all declinations appear on every page, the great-circle solution can always be extracted from the volume which covers the latitude of departure.

Great-circle solutions belong in one of the four following cases:

Case I—Latitudes of departure and destination of same name and initial great-circle distance less than 90°.

Enter the tables with latitude of departure as latitude argument (Same Name), latitude of destination as declination argument, and difference of longitude as local hour angle argument. If the respondents as found on a right-hand page do not lie below the C-S Line, Case III is applicable.

Extract the tabular altitude which subtracted from 90° is the desired great-circle distance. The tabular azimuth angle is the initial great-circle course angle.

Case II—Latitudes of departure and destination of contrary name and great-circle distance less than 90°.

Enter the tables with latitude of departure as latitude argument (Contrary Name) and latitude of destination as declination argument, and with the difference of longitude as local hour angle argument. If the respondents do not lie above the C-S Line on the right-hand page, Case IV is applicable.

Extract the tabular altitude which subtracted from 90° is the desired great-circle distance. The tabular azimuth angle is the initial great-circle course angle.

Case III—Latitudes of departure and destination of same name and great-circle distance greater than 90°.

Enter the tables with latitude of departure as latitude argument (Same Name), latitude of destination as declination argument, and difference of longitude as local hour angle argument. If the respondents as found on a right-hand page do not lie above the C-S Line, Case I is applicable.

Extract the tabular altitude which added to 90° gives the desired great-circle distance. The initial great-circle course angle is 180° minus the tabular azimuth angle.

Case IV—Latitudes of departure and destination of contrary name and great-circle distance greater than 90°.

Enter the tables with latitude of departure as latitude argument (Contrary Name), latitude of destination as declination argument and difference of longitude as local hour angle argument. If the respondents as found on a right-hand page do not lie below the C-S Line, Case II is applicable. If the DLo is in excess of 90°, the respondents are found on the facing left-hand page (See section C.4.).

Extract the tabular altitude which added to 90° gives the desired great-circle distance. The initial great-circle course angle is 180° minus the tabular azimuth angle.

The following two great-circle distance and course solutions illustrate Cases I and IV.

Case I

Required.—Distance and initial great-circle course from San Juan (18°28′N, 66°07′W) to Milford Haven (51°43′N, 5°02′W).

Solution.—(l) Case I is assumed to be applicable. Since the latitude of the point of departure, the latitude of the destination, and the difference of longitude (DLo) between the point of departure and destination are not integral degrees, the solution is effected from an adjusted point of departure or assumed position of departure chosen as follows: the latitude of the assumed position (AP) is the integral degrees of latitude nearest to the point of departure; the longitude of the AP is chosen to provide integral degrees of DLo. This AP, which should be within 30′ of the longitude of the point of departure, is at latitude 18°N, longitude 66°02′W. The DLo is 61°.

(2) Enter the tables with 18° as the latitude argument (Same Name), 61° as the LHA argument, and 51° as the declination argument.

(3) From page 124 extract the tabular altitude, altitude difference, and azimuth angle; interpolate altitude and azimuth angle for declination increment. The Dec. Inc. is the minutes that the latitude of the destination is in excess of the integral degrees used as the declination argument.

		ht (Tab. Hc)	d	Z
LHA 61°, Lat. 18° (Same),	Dec.51°	32° 01.6′	(−)11.9′	40.5°
Dec. Inc. 43′, d(−)11.9′	Tens	(−) 7.1		
	Units	(−) 1.4		
Interpolated for Dec. Inc.		31° 53.1′		C N39.6°E
Initial great-circle course from AP				Cn 039.6°
Great-circle distance from AP (90°−31°53.1′)				3486.9 n.mi.

(4) Using the graphical method for interpolating altitude for latitude and LHA increments, the course line is drawn from the AP in the direction of the initial great-circle course from the AP (039.6°). As shown in figure 12, a line is drawn from the point of departure perpendicular to the initial great-circle course line or its extension.

(5) The required correction, in units of minutes of latitude, for the latitude and DLo increments is the length along the course line between the foot of the perpendicular and the AP. The correction as applied to the distance from the AP is −18.4′; the great-circle distance is 3468 nautical miles.

(6) The azimuth angle interpolated for declination, LHA, and latitude increments is 39.8°; the initial great-circle course from the point of departure is 039.8°
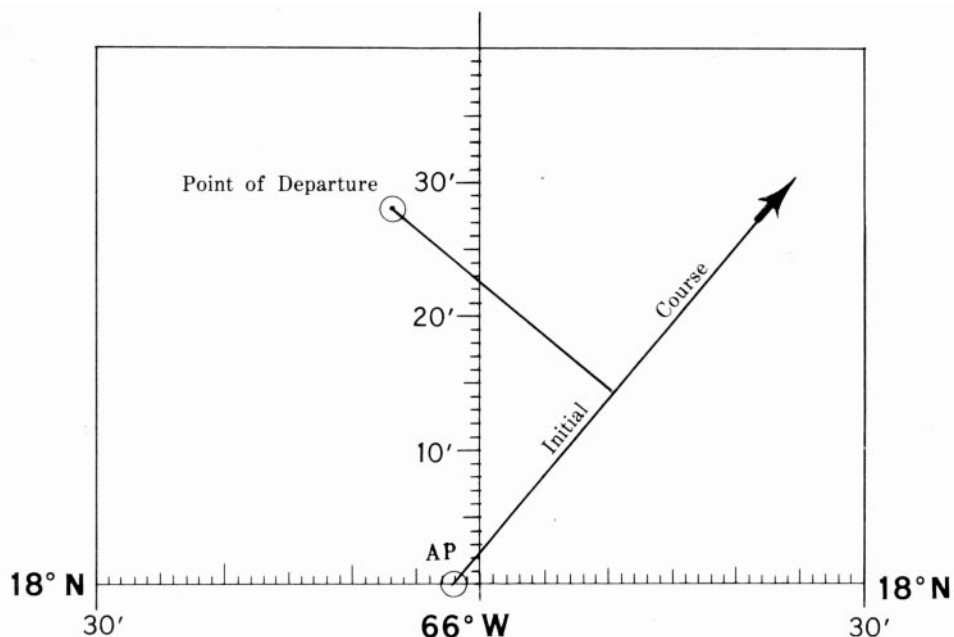
FIGURE 12

Case IV

Required.—Distance and initial great-circle course from Cape Moreton (27°02′S, 153°28′E) to Cape Flattery (48°24′N, 124°44′W).

Solution.—(1) Case IV is assumed to be applicable. Since the latitude of the point of departure, the latitude of the destination, and the difference of longitude (DLo) between the point of departure and destination are not integral degrees, the solution is effected from an adjusted point of departure or assumed position of departure chosen as follows: the latitude of the assumed position (AP) is the integral degrees of latitude nearest to the point of departure; the longitude of the AP is chosen to provide integral degrees of DLo. This AP, which should be within 30′ of the longitude of the point of departure, is at latitude 27°S, longitude 153°16′E. The DLo is 82°.

(2) Enter the tables with 27° as the latitude argument (Contrary Name), 82° as the LHA argument, and 48° as the declination argument.

(3) From page 349 extract the tabular altitude, altitude difference, and azimuth angle; interpolate altitude for Dec. Inc. as if the altitude were positive, adhering strictly to the sign given d. After interpolation regard the results as negative. Subtract tabular azimuth angle from 180°; interpolate for Dec. Inc.

		ht (Tab. Hc)	d	Z
LHA 82°, Lat. 27° (Contrary), Dec.48°		14°44.3′	(+)24.5′	43.2°
Dec. Inc. 24′, d(+)24.5′	Tens	(+) 8.0		180°− Z=136.8°
	Units	(+) 1.8		
Interpolated for Dec. Inc.		(−) 14°54.1′		C S137.2°E
Initial great-circle course from AP				Cn 042.8°
Great-circle distance from AP (90°+14°54.1′)				6294.1 n.mi.

(4) Using the graphical method for interpolating altitude for latitude and LHA increments, the course line is drawn from the AP in the direction of the initial great-circle course from the AP (042.8°). As shown in figure 13 a line is drawn from the point of departure perpendicular to the course line or its extension.

(5) The required additional correction, in units of minutes of latitude, for the latitude and DLo increments is the length along the course line between the foot of the perpendicular and the AP. The correction as applied to the distance from the AP is (−) 5.6′; the great-circle distance is 6288 nautical miles.

(6) The azimuth angle interpolated for declination, LHA, and latitude increments is 137.2°; the initial great-circle course from the point of departure is 042.8°.
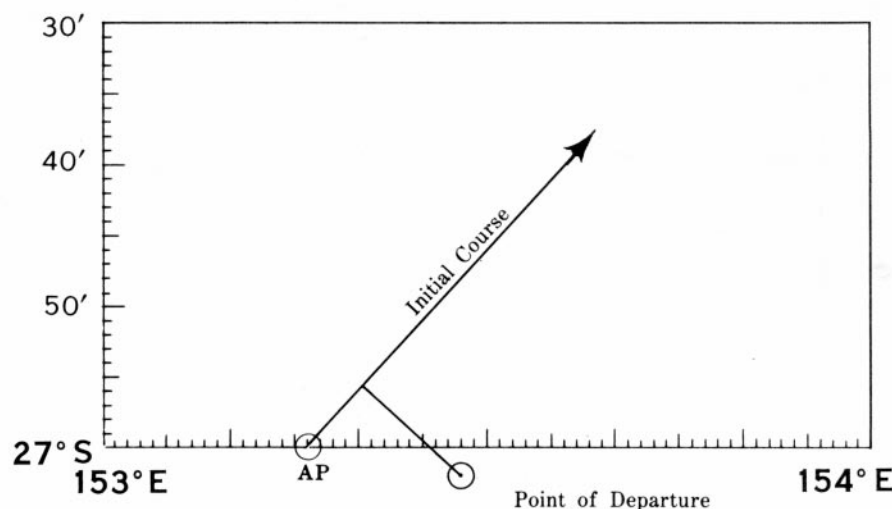
FIGURE 13

3. Points along Great Circle. If the latitude of the point of departure and the initial great-circle course angle are integral degrees, points along the great circle are found by entering the tables with the latitude of departure as the latitude argument (Same Name), the initial great-circle course angle as the LHA argument, and 90° minus distance to a point on the great circle as the declination argument. The latitude of the point on the great circle and the difference of longitude between that point and the point of departure are the tabular altitude and azimuth angle respondents, respectively.

Required.—A number of points at 300-mile intervals along the great circle from latitude 18°N, longitude 66°W when the initial great-circle course angle is N 40° E.

Entering the tables with latitude 18° (Same Name), LHA 40°, and with successive declinations of 85°, 80°, 75°, . . . the latitudes and differences in longitude, from 66°W, are found as tabular altitudes and azimuth angles, respectively.

Distance n. mi, (arc)	300(5°)	600(10°)	900(15°)	1200(20°)
Latitude	21.8°N	25.5°N	29.1°N	32.6°N
DLo	3.5°	7.1°	11.0°	15.1°
Longitude	62.5°W	58.9°W	55.0°W	50.9°W

Note.—If the respondents are abstracted from across the C-S line, the DLo is the supplement of the tabular azimuth angle; the tabular altitudes correspond to latitudes on the side of the equator opposite from the latitude of departure.

4. General Spherical Triangle Solutions. Of the six parts of the spherical astronomical triangle, these tables utilize three as entering arguments and tabulate two as respondents. The only remaining part of the triangle is the parallactic (or position) angle, which is the angle between a body's hour circle and its vertical circle. Values of the parallactic angle, not essential for navigation, have not been included in order to keep the tabulations to a minimum. However, the parallactic angle can be found through the simple interchange of arguments, thus effecting a complete solution. The applicable instructions are as follows:

(a) *When latitude and declination are of same name*, enter the tables with the appropriate local hour angle, with the declination as latitude argument of the same name and the latitude as declination argument, and extract the tabular azimuth angle as the parallactic angle.

(b) *When latitude and declination are of contrary name,* enter the tables with the appropriate local hour angle and with the declination as latitude argument of contrary name and the latitude as declination argument; the tabular azimuth angle is then the supplement of the parallactic angle (i.e., parallactic angle equals 180° minus the azimuth angle). This method generally requires the availability of all volumes of the series.

An approximate value of the parallactic angle, X, accurate enough for most navigational requirements, can be calculated directly from the formula, $\cos X = d/60'$, where d is the difference between successive tabular altitudes for the desired latitude, local hour angle and declination.

Within the limitations of the tabular precision and interval, the tabular data of these tables include the solution of any spherical triangle, given two sides and the included angle. When using the tables for the general solution of the spherical triangle, the use of latitude, declination, and altitude in the tables instead of their corresponding parts of the astronomical triangle must be kept in mind.

In general if any three parts of a spherical triangle are given, these tables can be used to find the remaining parts; this will sometimes mean searching through the volumes to find, for example, a particular altitude in a particular latitude and a given LHA in order to find the corresponding azimuth angle and declination.

5. Compass Error. One of the more frequent applications of sight reduction tables is their use in computing the azimuth of a celestial body for comparison with an observed azimuth in order to determine the error of the compass. In computing the azimuth of a celestial body, for the time and place of observation, it is normally necessary to interpolate the tabular azimuth angle as extracted from the tables for the differences between the table arguments and the actual values of declination, latitude, and local hour angle. The required triple interpolation of the azimuth angle is effected as follows:

(1)　The main tables are entered with the nearest integral values of declination, latitude, and local hour angle; for these arguments, a base azimuth angle is extracted.

(2)　The tables are reentered with the same latitude and LHA arguments but with the declination argument 1° greater or less than the base declination argument depending upon whether the actual declination is greater or less than the base argument. The difference between the respondent azimuth angle and the base azimuth angle establishes the azimuth angle difference (Z Diff.) for the increment of declination.

(3)　The tables are reentered with the base declination and LHA arguments but with the latitude argument 1° greater or less than the base latitude argument depending upon whether the actual (usually DR) latitude is greater or less than the base argument to find the Z Diff. for the increment of latitude.

(4)　The tables are reentered with the base declination and latitude arguments but with the LHA argument 1° greater or less than the base LHA argument depending upon whether the actual LHA is greater or less than the base argument to find the Z Diff. for the increment of LHA.

(5)　The correction to the base azimuth angle for each increment is $\text{Z Diff.} \times \dfrac{\text{Inc.}}{60'}$.

Example.—In DR Lat. 23°24.0′N, the azimuth of the Sun is observed as 072.5° pgc. At the time of the observation, the declination of the Sun is 20°13.8′N; the local hour angle of the Sun is 276°41.2′. The error of the gyrocompass is found as follows:

	Actual	Base Arguments	Base Z	Tab* Z	Z Diff.	Increments	Correction (Z Diff.×Inc.÷60)
Dec.	20°13.8′N	20°	73.9°	72.9°	-1.0°	13.8′	-0.2°
DR Lat.	23°24.0′N	23° (Same)	73.9°	74.1°	+0.2°	24.0′	+0.1°
LHA	276°41.2′	277°	73.9°	73.5°	-0.4°	18.8′	-0.1°

Base Z	73.9°
Corr.	(-) 0.2°
Z	N 73.7°E
Zn	073.7°
Zn pgc	072.5°
Gyro Error	1.2°E

Total Corr.　-0.2°

*Respondent for two base arguments and 1° change from third base argument, in vertical order of Dec., DR Lat., and LHA.

E. BACKGROUND

1. Accuracy of Tables. The tabular values as given in these tables have maximum and probable (50%) errors of ±0.05′ and ±0.025′ in altitude and ±0.05° and ±0.025° in azimuth angle.

The maximum error arising from the use of the Interpolation Table for the first-difference correction is ±0.14′, with a probable error of ±0.03′, when used for the interpolation of altitude for declination.

The maximum error arising from the use of the correction for second differences obtained from the Interpolation Table is ±0.12′ with a probable error of ± 0.03′.

When second differences are completely negligible, the maximum error of an interpolated altitude is ±0.19′ with a probable error of ±0.04′; when the second differences are not negligible and the second-difference correction is included in the interpolation, the maximum error of the calculated altitude will be ±0.31′ with a probable error of ±0.05′.

The largest value of the double-second difference when the value of d is not printed in italics is 3.9′, and if the correction for this value is neglected, an error of up to −0.24′ may be introduced into the computed altitude. But such an error is only possible when the altitude is greater than 60° and when the value of Dec. Inc. is close to 30′. The neglect of the second-difference correction when d is not printed in italics will rarely introduce an error as large as −0.2′.

For altitudes less than 86°, i.e., for zenith distances greater than 4°, interpolation of the tabular altitude for declination, utilizing both first and second differences and the Interpolation Table, may be made to within about 0.2′; linear interpolation for azimuth angle can be made to about 0.2°. Closer to the zenith, not only do second differences exceed the limits of the tables but higher differences are also significant.

When the body is in the zenith, its azimuth is indeterminate, that is when LHA is 0° and when latitude and declination are equal and have the Same Name. In these cases Z is tabulated as 90° or as one-half the preceding value. There are 91 of these cases.

When latitude is 90° and declination is 90°, the altitude is 90° for all hour angles. Here the value of Z tabulated is one-half the preceding value. There are 182 of these cases, two of which are included in the previous set. In the above cases the tabulated azimuth angles are the mathematical limits of the azimuth angle when the limit is approached in a specified direction.

In the special cases when the latitude is 90°, i.e., at the poles, all directions from the North Pole are south and from the South Pole are north; the criterion adopted in these cases has been to tabulate the azimuth as equal to 180° minus LHA, i.e., the directions are tabulated as the angular directions from the lower branch of the Greenwich Meridian. There are 90×180 of these cases not included in the previous sets.

2. Computation formulas. For latitude (L), declination (d) and local hour angle (LHA), the altitude (Hc) and the azimuth angle (Z) were calculated from the following formulas:

$$\sin Hc = \sin L \sin d + \cos L \cos d \cos LHA$$

$$\tan Z = \frac{\cos d \sin LHA}{\cos L \sin d - \sin L \cos d \cos LHA}$$

All values of altitude within 1°30′ of the zenith were recalculated using a more appropriate formula because determination of these high altitudes from their sines with only nine figures could introduce errors of the order of 0.0005′, which would sometimes affect the rounding off of the altitude to 0.1′. The formula used is equivalent to:

$$\sin^2 \tfrac{1}{2} z = \cos^2 \tfrac{1}{2} LHA \sin^2 \tfrac{1}{2} (L\text{-}d) + \sin^2 \tfrac{1}{2} LHA \cos^2 \tfrac{1}{2} (L\text{+}d), \text{ where z is the zenith distance.}$$

F. GLOSSARY

Altitude—the arc of a vertical circle between the horizon and a point or body on the celestial sphere. Altitude as measured by a sextant is called sextant altitude (**hs**). Sextant altitude corrected only for inaccuracies in the reading (instrument, index, and personal errors, as applicable) and inaccuracies in the reference level (principally dip) is called apparent altitude (**ha**). After all corrections are applied, it is called corrected sextant altitude or observed altitude (**Ho**). An altitude taken directly from a table is called a tabular or tabulated altitude (**ht**). Tabular altitude as interpolated for declination, latitude, and LHA increments as required is called computed altitude (**Hc**).

Altitude Difference (d)—the first difference between successive tabulations of altitude in a latitude column of these tables.

Argument—one of the values used for entering a table or diagram.

Assumed (or Chosen) Latitude (aL), Assumed (or Chosen) Longitude (aλ)—geographical coordinates assumed to facilitate sight reduction.

Assumed Position (AP)—a point at which an observer is assumed to be located.

Azimuth (Zn)—the horizontal direction of a celestial body or point from a terrestrial point; the arc of the horizon, or the angle at the zenith, between the north part of the celestial meridian or principal vertical circle and a vertical circle through the body or point, measured from 000° at the north part of the principal vertical circle clockwise through 360°.

Azimuth Angle (Z)—the arc of the horizon, or the angle at the zenith, between the north part or south part of the celestial meridian, according to the elevated pole, and a vertical circle through the body or point, measured from 0° at the north or south reference eastward or westward through 180° according to whether the body is east or west of the local meridian. It is prefixed N or S to agree with the latitude and suffixed E or W to agree with the meridian angle.

Celestial Equator—the primary great circle of the celestial sphere, everywhere 90° from the celestial poles; the intersection of the extended plane of the equator and the celestial sphere. Also called EQUINOCTIAL.

Celestial Horizon—that circle of the celestial sphere formed by the intersection of the celestial sphere and a plane through the center of the Earth and perpendicular to zenith-nadir line.

Celestial Meridian—on the celestial sphere, a great circle through the celestial poles and the zenith. The expression usually refers to the upper branch, that half from pole to pole which passes through the zenith.

Course Angle—course measured from 0° at the reference direction clockwise or counterclockwise through 180°. It is labeled with the reference direction as a prefix and the direction of measurement from the reference direction as a suffix. Thus, course angle S21°E is 21° east of south, or true course 159°.

Course Line—the graphic representation of a ship's course.

Declination (Dec.)—angular distance north or south of the celestial equator; the arc of an hour circle between the celestial equator and a point on the celestial sphere, measured northward or southward from the celestial equator through 90°, and labeled N or S (+ or −) to indicate the direction of measurement.

Declination Increment (Dec. Inc.)—in sight reduction, the excess of the actual declination of a celestial body over the integral declination argument.

Double-Second Difference (DSD)—the sum of successive second differences. Because second differences are not tabulated in these tables, the DSD can be formed most readily by subtracting, algebraically, the first difference immediately above the tabular altitude difference (d) corresponding to the entering arguments from the first difference immediately below. The result will always be a negative value.

Ecliptic—the apparent annual path of the Sun among the stars; the intersection of the plane of the Earth's orbit with the celestial sphere. This is a great circle of the celestial sphere inclined at an angle of about 23°27′ to the celestial equator.

Elevated Pole (Pn or Ps)—the celestial pole above the observer's horizon, agreeing in name with the observer's latitude.

First Difference—the difference between successive tabulations of a quantity.

First Point of Aries (Υ)—that point of intersection of the ecliptic and the celestial equator occupied by the Sun as it changes from south to north declination on or about March 21. Also called VERNAL EQUINOX.

Geographical Position (GP)—the point where a line drawn from a celestial body to the Earth's center passes through the Earth's surface.

Great Circle—the intersection of a sphere and a plane through its center.

Great-Circle Course—the direction of the great circle through the point of departure and the destination, expressed as angular distance from a reference direction, usually north, to the direction of the great circle. The angle varies from point to point along the great circle. At the point of departure it is called INITIAL GREAT-CIRCLE COURSE.

Greenwich Hour Angle (GHA)—angular distance west of the Greenwich celestial meridian; the arc of the celestial equator, or the angle at the celestial pole, between the upper branch of the Greenwich celestial meridian and the hour circle of a point on the celestial sphere, measured westward from the Greenwich celestial meridian through 360°.

Hour Circle—on the celestial sphere, a great circle through the celestial poles and a celestial body or the vernal equinox. Hour circles are perpendicular to the celestial equator.

Intercept (a)—the difference in minutes of arc between the computed and observed altitudes (corrected sextant altitudes). It is labeled T (toward) or A (away) as the observed altitude is greater or smaller than the computed altitude; Hc greater than Ho, intercept is away (A); Ho greater than Hc, intercept is toward (T).

Line of Position (LOP)—a line indicating a series of possible positions of a craft, determined by observation or measurement.

Local Hour Angle (LHA)—angular distance west of the local celestial meridian; the arc of the celestial equator, or the angle at the celestial pole, between the upper branch of the local celestial meridian and the hour circle of a celestial body or point on the celestial sphere, measured westward from the local celestial meridian through 360°.

Meridian Angle (t)—angular distance east or west of the local celestial meridian; the arc of the celestial equator, or the angle at the celestial pole, between the upper branch of the local celestial meridian and the hour circle of a celestial body, measured eastward or westward from the local celestial meridian through 180°, and labeled E or W to indicate the direction of measurement.

Nadir (Na)—that point on the celestial sphere 180° from the observer's zenith.

Name—the labels N and S which are attached to latitude and declination are said to be of the same name when they are both N or S and contrary name when one is N and the other is S.

Navigational Triangle—the spherical triangle solved in computing altitude and azimuth and great-circle sailing problems. The celestial triangle is formed on the celestial sphere by the great circles connecting the elevated pole, zenith of the assumed position of the observer, and a celestial body. The terrestrial triangle is formed on the Earth by the great circles connecting the pole and two places on the Earth: the assumed position of the observer and geographical position of the body for celestial observations, and the point of departure and destination for great-circle sailing problems. The term astronomical triangle applies to either the celestial or terrestrial triangle used for solving celestial observations.

Polar Distance (p)—angular distance from a celestial pole; the arc of an hour circle between a celestial pole, usually the elevated pole, and a point on the celestial sphere, measured from the celestial pole through 180°.

Prime Meridian—the meridian of longitude 0°, used as the origin for measurement of longitude.

Prime Vertical—the vertical circle through the east and west points of the horizon.

Principal Vertical Circle—the vertical circle through the north and south points of the horizon, coinciding with the celestial meridian.

Respondent—the value in a table or diagram corresponding to the entering arguments.

Second Difference—the difference between successive first differences.

Sidereal Hour Angle (SHA)—angular distance west of the vernal equinox; the arc of the celestial equator, or the angle at the celestial pole, between the hour circle of the vernal equinox and the hour circle of a point on the celestial sphere, measured westward from the hour circle of the vernal equinox through 360°.

Sight Reduction—the process of deriving from a sight (observation of the altitude, and sometimes also the azimuth, of a celestial body) the information needed for establishing a line of position.

Small Circle—the intersection of a sphere and a plane which does not pass through its center.

Vertical Circle—on the celestial sphere, a great circle through the zenith and nadir. Vertical circles are perpendicular to the horizon.

Zenith (Z)—that point on the celestial sphere vertically overhead.

Zenith Distance (z)—angular distance from the zenith; the arc of a vertical circle between the zenith and a point on the celestial sphere.

G. EXAMPLE SIGHT REDUCTIONS

Example—On September 9, 1974, the 1932 dead reckoning position of a ship is lat. 14°45′ N, long. 30°00′ W. The ship is on course 225°, speed 20 knots. Observations are made from a height of eye of 31 feet using a sextant having an index error of (+) 1.0′ as indicated below. Determine the 1932 fix.

Body	Zone Time	Sextant Altitude	SHA	Declination
Arcturus	19^{h}20^{m}03^s	28°29.5′	146°22.4′	19°18.9′ N
Antares	19^{h}25^{m}58^s	37°57.4′	113°02.0′	26°22.7′ S
Rasalhague	19^{h}32^{m}01^s	72°38.2′	96°34.0′	12°34.9′ N

|	**ARCTURUS**		**ANTARES**		**RASALHAGUE**
GMT (Sept. 9) -	21^{h}20^{m}03^s		21^{h}25^{m}58^s		21^{h}32^{m}01^s
GHA ♈ for 21^h GMT- - - - - - - - - - - - - - - - -	303°31.5′		303°31.5′		303°31.5′
Increments- -20^{m}03^s	5°01.6′	25^{m}58^s	6°30.6′	32^{m}01^s	8°01.6′
SHA -	146°22.4′		113°02.0′		96°34.0′
GHA☆ -	94°55.5′		63°04.1′		48°07.1′
aλ- -	29°55.5′W		30°04.1′W		30°07.1′W
LHA☆ -	65°00.0′		33°00.0′		18°00.0′
Dec. -	19°18.9′N		26°22.7′S		12°34.9′N
Dec. Inc. -	18.9′		22.7′		34.9′
aL -	15°00.0′N		15°00.0′N		15°00.0′N
ht (Tab. Hc) -	28°03.0′		37°55.6′		72°14.9′
d and correction - - - - - - - - - - - - - - - -(+) 7.3′	(+) 2.3′	(−) 44.9′	(−) 17.0′	(+) 10.6′	(+) 6.6′
Hc -	28°05.3′		37°38.6′		72°21.5′
Ho -	28°21.3′		37°49.8′		72°31.5′
a -	16.0T		11.2T		10.0T
Z and Zn - - - - - - - - - - - - - - - - - - - N75.8°W	284.2°	N141.9°W	218.1°	N95.7°W	264.3°
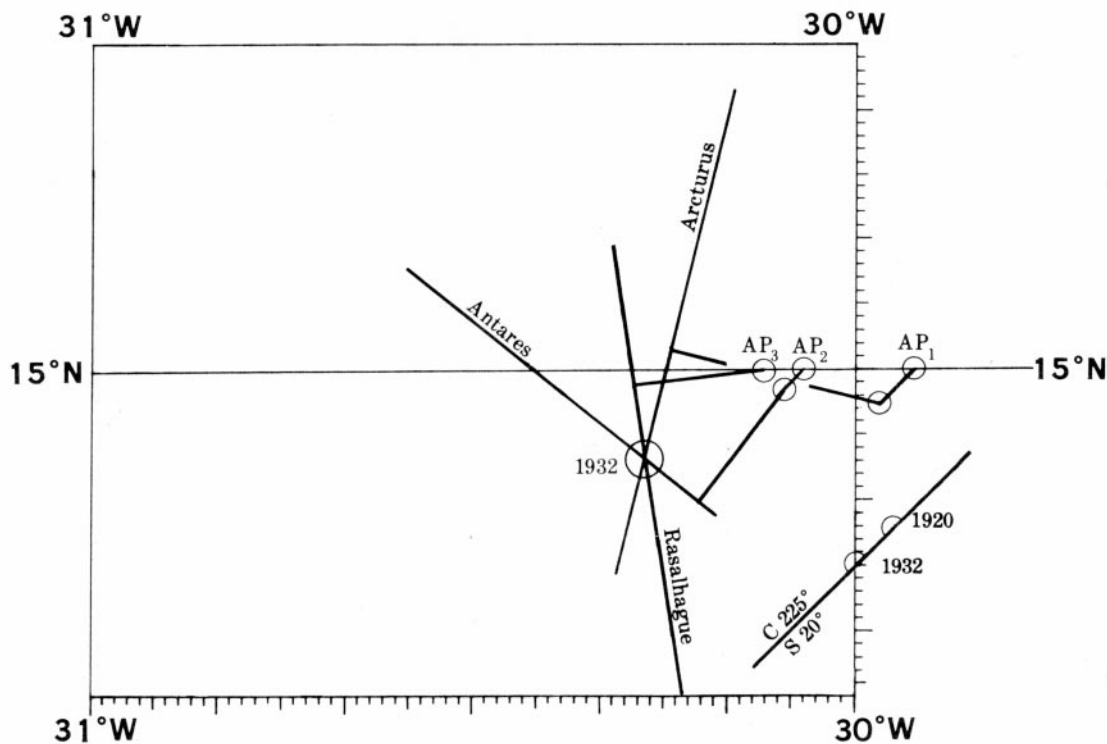
FIGURE 14

Note.—In figure 14 the assumed position of the Arcturus line of position is advanced 4.0 miles for a 12-minute run and the assumed position of the Antares line of position is advanced 2.0 miles for a 6-minute run, both in the direction of the course 225°, to obtain a fix at the time of the Rasalhague sight. Each azimuth angle is interpolated for declination increment. The interpolation of the tabular altitude of the Rasalhague sight includes a DSD correction of (+) 0.4′.